Ch11: AC Power Analysis

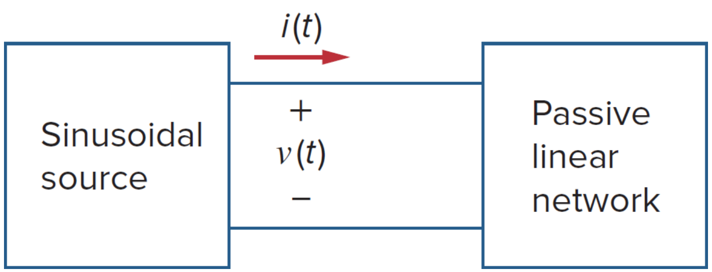
Instantaneous and Average Power

Instantaneous Power

Instantaneous Power $p(t)$ (in watts) is the power at any instant in time. It is the rate at which an element absorbs power, which is the product of the instantaneous voltage $v(t)$ across the element and the instantaneous current $i(t)$ through it.

$$p(t) = v(t)i(t)$$

- Consider the general case of instantaneous power absorbed by an arbitrary combination of circuit elements under sinusoidal excitation.



$$v(t) = V_m \cos(\omega t + \theta_v)$$
$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\boxed{p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)}$$

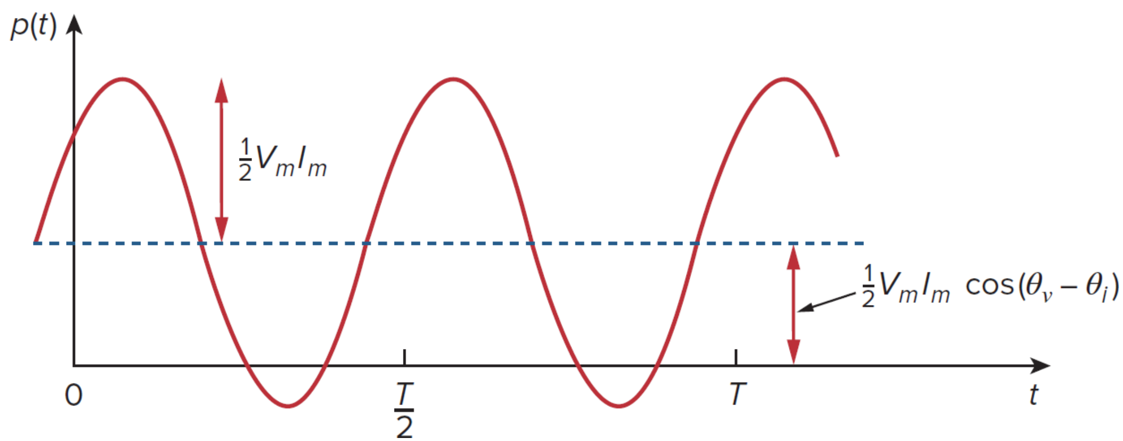
Instantaneous Power

Note that the instantaneous power $p(t)$ has two parts:

- The first part is constant, depending on the phase difference between the voltage and current,
- The second part is sinusoidal with a frequency twice that of the voltage and current (2ω).

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

A sketch of the instantaneous power $p(t)$:

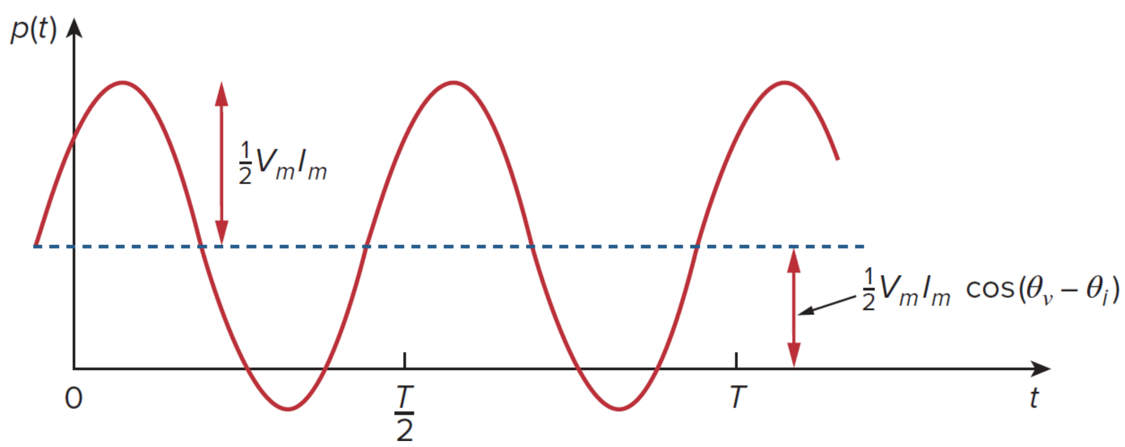


$T = 2\pi/\omega$

Instantaneous Power

We observe that

- $p(t)$ is periodic, $p(t) = p(t + T_0)$, and has a period of $T_0 = T\ /\ 2$, since its frequency is twice that of voltage or current. Note: $T = 2\pi/\omega$.
- $p(t)$ is **positive** for some part of each cycle (where power is absorbed by the circuit) and **negative** for the rest of the cycle (where power is absorbed by the source; that is, power is transferred from the circuit to the source). This is possible because of the storage elements (e.g., capacitors and inductors) in the circuit.



Average Power

The instantaneous power changes with time and is therefore difficult to measure. The **Average Power** is more convenient to measure.

The **Average Power**, in watts, is the average of the instantaneous power over one period.

$$P = \frac{1}{T_0}\int_0^{T_0} p(t)dt = \frac{1}{T}\int_0^{T} p(t)dt$$

$$= \frac{1}{T}\int_0^{T} \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)dt + \frac{1}{T}\int_0^{T} \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)dt$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\cancelto{T}{\int_0^{T} dt} + \frac{1}{2}V_m I_m \frac{1}{T}\cancelto{0}{\int_0^{T} \cos(2\omega t + \theta_v + \theta_i)dt}$$

$$\boxed{P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)}$$

(Average Power P does not depend on time)

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Average Power

- To find the instantaneous power, we must necessarily have $v(t)$ and $i(t)$ in the time domain.
- But we can find the average power when voltage and current are expressed in the time domain ($P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$) or when they are expressed in the frequency domain.

Let the phasor forms of $v(t)$ and $i(t)$ be $\mathbf{V} = V_m \angle \theta_v$ and $\mathbf{I} = I_m \angle \theta_i$, respectively. Thus, the average power P will be

$$P = \frac{1}{2}\mathrm{Re}[\mathbf{V}\mathbf{I}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

Note: $\mathbf{I}^*$ is complex conjugate of $\mathbf{I}$. For example, if $\mathbf{I} = x + jy$, then $\mathbf{I}^* = x - jy$.

Resistive versus Reactive

Consider two special cases:

- When $\theta_v = \theta_i$, the voltage and current are in phase and the circuit/load is purely resistive (R):

$$P = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R = \frac{1}{2} (R\mathbf{I})(\mathbf{I}^*) = \frac{1}{2} |\mathbf{I}|^2 R$$

Therefore, a <u>purely resistive</u> circuit/load (R) <u>absorbs power at all times</u>.

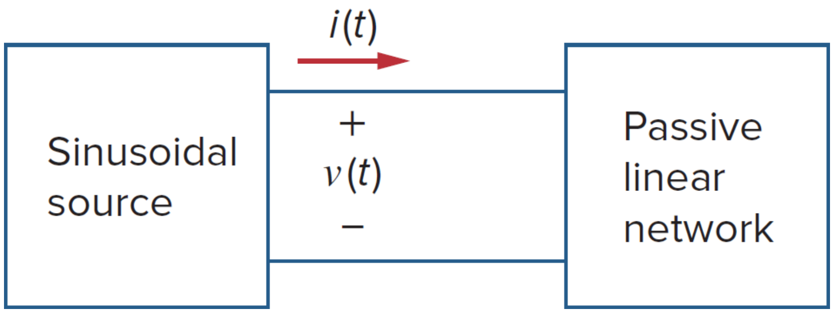
- When $\theta_v - \theta_i = \pm 90°$, the circuit/load is purely reactive (L or C):

$$P = \frac{1}{2} V_m I_m \cos 90° = 0$$

Therefore, a <u>purely reactive</u> circuit/load (L or C) <u>absorbs no average power</u>.

Example

Given that $v(t) = 120\cos(377t + 45°)$ V and $i(t) = 10\cos(377t - 10°)$ A, find the instantaneous power and the average power absorbed by the passive linear network of the following figure.

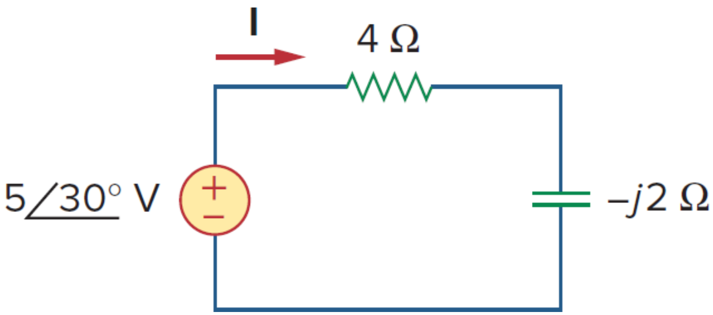


Example

Calculate the average power absorbed by an impedance $\mathbf{Z} = 30 - j70\ \Omega$ when a voltage $\mathbf{V} = 120\angle 0°$ is applied across it.

Example

For the circuit shown, find the average power supplied by the source and the average power absorbed by the resistor.



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Example

Determine the average power generated by each source and the average power absorbed by each passive element in the circuit.