Ch1: Error Analysis in Measurements

Introduction

Uncertainty

Uncertainty (u_x or Δx) is a numerical estimate of the **possible range of the error** in a measurement. It establishes a bound in which the measured value is allowed to fall. A typical expression for reporting a measurement value:

$$x' = \bar{x} \pm u_x \;\; (P\%)$$

- x' is the **true value** of the parameter being measured.
- $\bar{x}$ is the **nominal value** (or the **best estimate** or, the **most probable** value).
- u_x or Δx is the **uncertainty** (or error, or range within which the measured value may vary) and is always positive. In this form, it has the same units as $\bar{x}$.
- $P\%$ is the **confidence interval**, which represents how often the measured value will fall within the reported range (95%, 68%, 99%, and 50%).

(The most common)

Alternatively, the uncertainty can be expressed as a percent of the nominal value ($v\%$):

$$x' = \bar{x} \pm v\% \;\; (P\%)$$

Uncertainty

Note: Reporting a measurement without an uncertainty bound is an incomplete statement of the measurement value.

Example:
To simply say that the temperature of an oven is 154 ºC is not necessarily a useful value. For example, if the oven temperature is 154 ± 2 ºC, and the process in the oven requires a temperature of 154 ± 0.5 ºC, then the oven temperature is unacceptable. On the other hand, if the required temperature is 154 ± 5 ºC, then the temperature is acceptable.

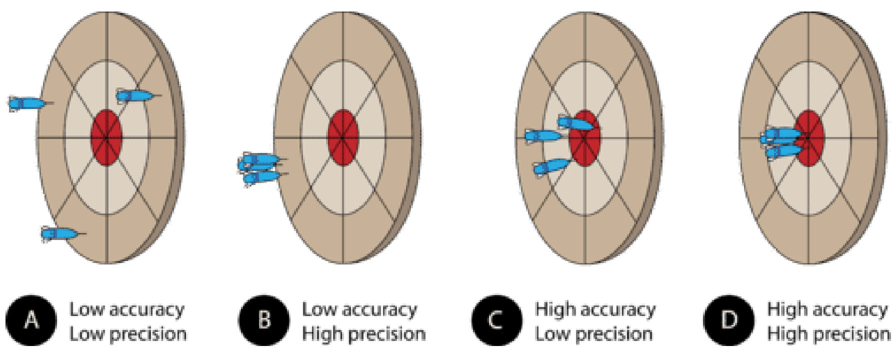
Note: By convention, if a measurement is stated without any uncertainty, it would be presumed that the true value of the measurement lies between the measured value $\pm$ half the place of the last significant digit present in the measurement value.

$$x = 1.27 \pm 0.005$$

Accuracy and Precision in Measurements

Accuracy and **Precision** indicate the **quality** of a measurement.

Accuracy refers to how close the measured value is to the true value.
Precision (or repeatability) refers to how repeatable the independent measurement of a particular constant value is (or how much variation there is from measurement to measurement).



Example:
Consider the following measurements (each consists of four readings) made on a system to measure π= 3.14159…

Case 1:	2.1415	2.1414	2.1416	2.1415
Case 2:	3.12	3.17	3.09	3.16
Case 3:	3.1415	3.1413	3.1412	3.1417

Case 1 is precise, but not accurate. **Case 2** is accurate, but not very precise. **Case 3** is both accurate and precise.

Three Ways of Expressing Accuracy

Absolute Error: $\varepsilon = \text{True Value} - \text{Measured Value}$ ε= 32.00 - 31.91 = 0.09 kg (same units as measurement value)

Relative Error: $\varepsilon_r = \dfrac{\text{True Value} - \text{Measured Value}}{\text{True Value}}$ ε_r= 0.003 or 0.3% (no unit)

Relative Accuracy: $A = 1 - \varepsilon_r$ A= 99.7% (no unit)

Values of A close to unity imply the measured value is accurate, i.e., the measured value is close to the true value.

- **Note**: ε has the same units as the measurement value, however ε_r and A has no units.
- **Note**: The sign of ε and ε_r indicates whether the measured value is greater than, or less than, the true value.

Accuracy of a measurement is determined by placing a **known** input (**True Value**) into the system and recording the output. This process is part of the process of **calibration** and the known input is called a **standard**.

Measurement Error Types

Three basic types of measurement error:

1. Bias or Systematic Error
2. Random or Precision Error
3. Illegitimate Error

} These **Errors** introduce **Measurement Uncertainty**.

1. Bias or Systematic Error

These are **fixed or constant values of the error** in a given set of measurements. In most cases they can be accounted by **calibrating** the experiment.

Example:
Measuring your height with your shoes on!
Error: The height of your shoes.

❖ Sources of Bias error include:

1. Errors during calibration,
2. Loading errors, i.e., the measurement system alters the value of the original system,
3. Unaccounted for effects that remain <u>constant</u> with time.

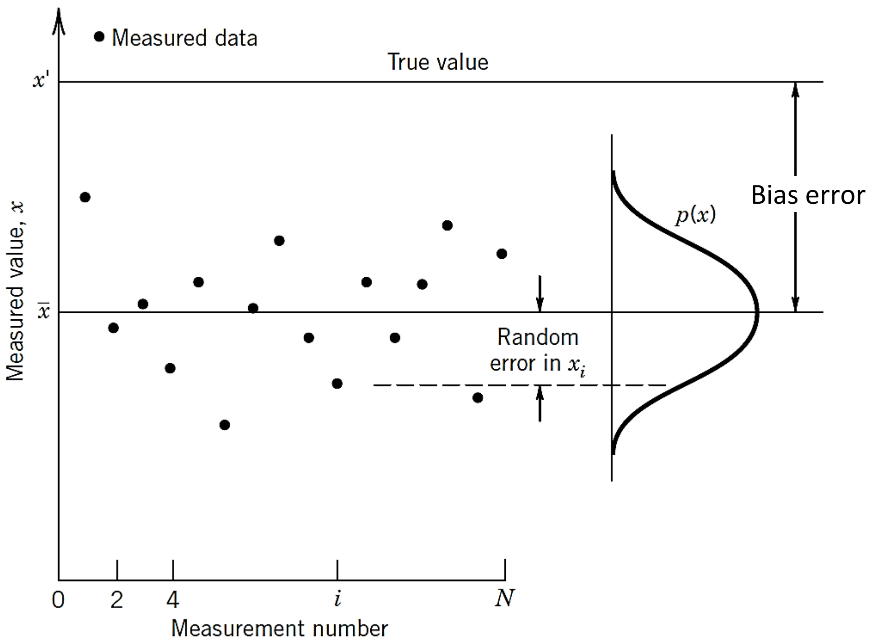
Example: Forgetting to account for the weight of the wax paper when measuring a small amount of chemical on an analytic balance.

2. Random or Precision Error

These errors are **random in nature** due to

1. Varying environmental conditions (e.g., changing room temperature and pressure),
2. Insufficient sensitivity of the measuring system,
3. Drift and fluctuation in the measurement system itself.

- This error can be treated by **statistical analysis**.
- The relationship between bias and random errors:



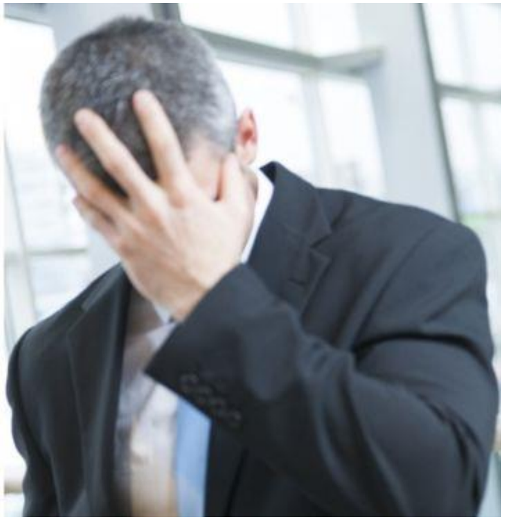
Note the **constant offset** produced by the bias error, and the **spread of the data** about $\bar{x}$ due to random error.

3. Illegitimate Error

These errors occur mainly due to **oversight** (or carelessness!) and consists of

1. Blunders or mistakes (e.g., reading mm as cm or inches),
2. Computational errors (e.g., wrong formula or mixed units),
3. Incorrect system operation.

- These errors are **not** acceptable as reasons for discrepancies between measured and expected values.
- If illegitimate errors are the cause of the error, the best solution is to **repeat** the experiment to avoid them.



Probability and Statistics

Probability and Statistics

Some Definitions:

- **Statistics** is the analysis of data or events that have already happened.
- **Probability** is the estimation of how likely future events are expected to occur, based on their history.
- **Population** (N) is the entire set of objects, events or measurements.
- **Sample** (n) is a representative subset of the population, which are randomly selected.

$$n \ll N$$

Example:

Consider a factory that produces 10,000 light bulbs per day. The population would be the entire collection of light bulbs for a particular day. The sample would be a representative subset, e.g., say 50 light bulbs that are randomly picked for testing. Here $N = 10{,}000$ and $n = 50$.

Central Tendency

In many engineering applications, systems have a tendency to gravitate towards a central value. **Mean**, **Median**, and **Mode** are the common measures of **central tendency**.

- **Mean** (or **Average**) is calculated by adding each individual value in the measurement set and then dividing by the total number of elements.

Population Mean: $$\mu = \sum_{i=1}^{N} \frac{x_i}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Sample Mean: $$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

In general, they are not the same.

- **Median** is the value that lies in the middle of a list of values after being sorted from lowest to highest value.

For odd number of values, the one in the middle: 2.3, 3.1, <u>3.9</u>, 4.7, 5.6

For even number of values, average the two centermost values : 2.4, 3.1, <u>3.9</u>, <u>4.6</u>, 5.6, 6.8
(4.25)

- **Mode** is the value that occurs most frequently. If two or more values occur with equal frequency, there are two or more modes.

Uncertainty Analysis

Uncertainty Analysis

Uncertainty Analysis (or **Error Analysis**) is the procedure by which uncertainties in measured quantities are ascertained, and the relationship between these measured values and the reported value is established.

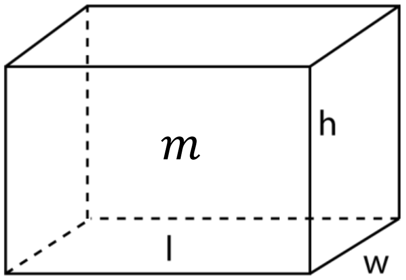
Two key steps of Uncertainty Analysis:

1. Determining the Uncertainty of the **Measured Input Variables**.
2. Propagating the Error in the Measured Values to **Calculated Values** (Output Variables).

Example:

Calculation of the density of a rectangular piece of wood:

- The input variables length l, width w, height h, and mass m are measured.
- The output variables volume V and density ρ are calculated from these input variables: $V = lwh,\ \rho = m/V$



Analysis of uncertainty in the ρ measurement involves 1) assessing the uncertainty in l, w, h, and m, and then 2) propagating the error to the calculated variables V and then ρ.

Uncertainty in Measured Input Variables

There are two main sources of uncertainty in measured input variables:

(1) **Instrument Uncertainty**

(2) **Measurement Uncertainty**

Instrument Uncertainty arises from the measurement system itself, and represents errors associated with the equipment responsible for making the measurement.

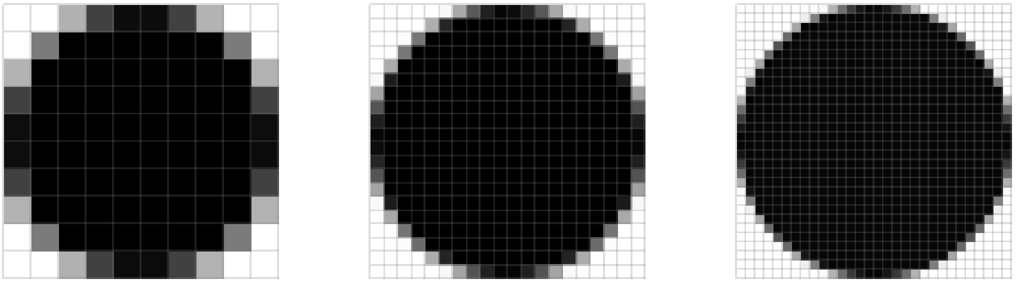
❖ Two basic kinds of instrument uncertainty:

- **Resolution Uncertainty**
- **Manufacturer's Uncertainty**

Instrument Uncertainty: Resolution Uncertainty

Resolution Uncertainty (u_0) is the inability of the measurement device to resolve to an infinite number of digits and it is represented as one-half of the instrument resolution:

$$u_0 = \pm \frac{1}{2}(\text{instrument resolution})$$



Example:

If the instrument resolution is 0.1 °C, and its resolution uncertainty is ±0.05 °C.

Instrument Uncertainty: Manufacturer's Uncertainty

Manufacturer's Uncertainty (u_c) refers to specific errors in the instrument which are reported by the manufacturer, including linearity, hysteresis, repeatability, etc.

If u_j ($j = 1,2,3, \ldots$) are the individual manufacturer's uncertainties, the total manufacturer's uncertainty is combined using the **Root Sum Square** (**RSS**) method as

$$u_c = [u_1^2 + u_2^2 + u_3^2 + \cdots]^{1/2}$$

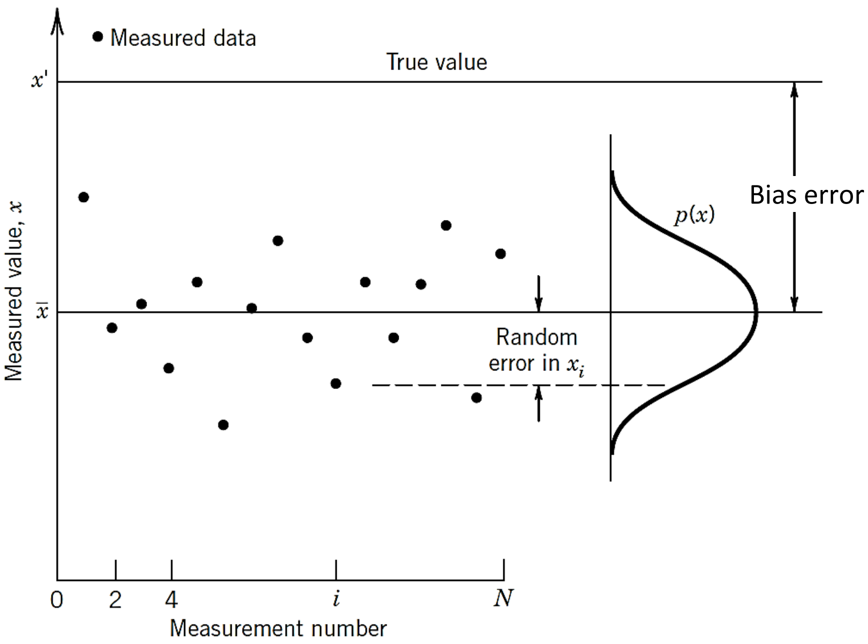
Finally, the **total instrument uncertainty** is obtained by combing the resolution and manufacturer's uncertainties as

$$u_{\text{inst}} = [u_0^2 + u_c^2]^{1/2}$$

Measurement Uncertainty

Assume that n measurements $(x_i, i = 1, \dots, n)$ are collected. Both **Bias Errors** and **Random Errors** can be present in measured variables.

- **Bias Errors** must be determined through **calibration**.
- **Random Errors** can be determined using statistics as **uncertainty**.



Some Definitions

Mean or **Average** ($\bar{x}$):

$$\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j$$

Sample Standard Deviation (S_x or σ) :

$$S_x = \sigma = \left[\frac{1}{n-1}\sum_{j=1}^{n}\left(x_j - \bar{x}\right)^2\right]^{1/2}$$

Mean Deviation:

$$\text{Mean Deviation} = \frac{1}{n}\sum_{j=1}^{n}\left|x_j - \bar{x}\right|$$

Variance:

$$\text{Variance} = (S_x)^2 = \sigma^2$$

The Sample Standard Deviation, Mean Deviation, and Variance provides a measure of how far numbers within a set trend from each other and can be used to provide a better understanding of how the set is internally distributed.

Two types of measurement uncertainty:

- **Next Measurement Uncertainty** (for a <u>single</u> future measurement)
- **Sample Mean Uncertainty** (for a <u>series</u> of measurements)

Next Measurement Uncertainty

It is sometimes desirable to know what the range of an additional measurement $(n+1)$ is expected to be, after making n measurements. This expected range is expressed as

$$x_{n+1} = \bar{x} \pm u_x \ \ (P\%)$$

where $\bar{x}$ is the mean and u_x is the uncertainty of the next measurement with confidence interval P.

Example:

1.34 ± 0.23 (95%) means about 95% of the time, the next measurement will fall between 1.11 and 1.57, and about 5% of the time, the measurement will fall outside of this region.

Next Measurement Uncertainty

The calculation of the next measurement uncertainty u_x depends on how many measurements are made, with the cutoff being about 60 samples.

- For **Large Sample Size** (the number of measurements $n > 60$), infinite statistics can be used to obtain the uncertainty directly:

$$u_x = S_x \qquad (68\%)$$
$$u_x = 1.96 S_x \qquad (95\%)$$
$$u_x = 2.58 S_x \qquad (99\%)$$

- For **Small Sample Size** ($n \leq 60$), the uncertainty increases somewhat due to the limited number of samples available for the statistics. the so-called **Student *t*-distribution** is used to determine the uncertainty

$$u_x = t_{v,P} \cdot S_x$$

$t_{v,P}$ is Student's t-distribution which is obtained from the table in the next slide.
$v = n - 1$ is number of degrees of freedom (ranging from 1 to 60).
P is the desired confidence interval (in %).

Student's t-Distribution Table

ν	t_{50}	t_{90}	t_{95}	t_{99}
1	1.000	6.314	12.706	63.657
2	0.816	2.920	4.303	9.925
3	0.765	2.353	3.182	5.841
4	0.741	2.132	2.770	4.604
5	0.727	2.015	2.571	4.032
6	0.718	1.943	2.447	3.707
7	0.711	1.895	2.365	3.499
8	0.706	1.860	2.306	3.355
9	0.703	1.833	2.262	3.250
10	0.700	1.812	2.228	3.169
11	0.697	1.796	2.201	3.106
12	0.695	1.782	2.179	3.055
13	0.694	1.771	2.160	3.012
14	0.692	1.761	2.145	2.977
15	0.691	1.753	2.131	2.947
16	0.690	1.746	2.120	2.921
17	0.689	1.740	2.110	2.898
18	0.688	1.734	2.101	2.878
19	0.688	1.729	2.093	2.861
20	0.687	1.725	2.086	2.845
21	0.686	1.721	2.080	2.831
30	0.683	1.697	2.042	2.750
40	0.681	1.684	2.021	2.704
50	0.680	1.679	2.010	2.679
60	0.679	1.671	2.000	2.660
∞	0.674	1.645	1.960	2.576

Note: As $v \to \infty$, the values of $t_{v,P}$ approach the values in equation for large sample size.

Example

Consider the following set of pressure readings made on a compressed air system (in psi):

20.3, 21.2, 19.8, 19.7, 17.9, 18.2, 22.3, 21.0, 19.8

What is the expected range of a subsequent measurement for a 95% probability?

Sample Mean Uncertainty

Let's say you collect a random sample and calculate the sample mean and standard deviation. Say you then collected a new random sample and calculated its mean and standard deviation. You would find that, in general, each sample yields a slightly different sample mean and sample standard deviation. This is not surprising, since we are picking different subsets of the entire population each time. **Sample Mean Uncertainty** determine how close the mean of a collection of n distinct measurements is to the true (population) mean.

The range of the true value of the population mean (μ) is expressed as

$$\mu = \bar{x} \pm t_{v,P} \cdot S_{\bar{x}} \quad (P\%)$$

where $\bar{x}$ is the sample mean, $t_{v,P}$ is Student's t-distribution, P is confidence interval, and $S_{\bar{x}}$ is standard deviation of mean and is defined as:

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

(S_x is sample standard deviation)

Note that $u_{\text{meas}} = t_{v,P} \cdot S_{\bar{x}}$ is the **Sample Mean Uncertainty**.

Example

Consider the previous example of the pressure readings. Estimate the bounds of the population mean, based on the sample statistics.

Total Uncertainty

The Total Uncertainty is obtained by combing the instrument uncertainty and measurement uncertainty (sample mean uncertainty) as

$$u_{\text{tot}} = \left[u_{\text{inst}}^2 + u_{\text{meas}}^2\right]^{1/2}$$

Error Propagation

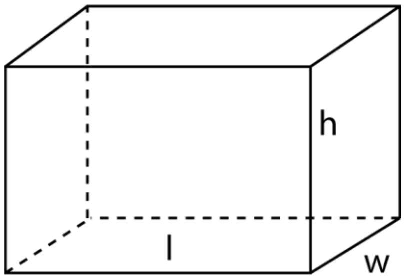
Error Propagation

In most measurements, the measured values (input variables) differs from the reported values (output variables). Hence, the **error propagation** from the input variables to the output variables should be determined.

Example:

Calculation of the density of a rectangular piece of wood:

- The input variables length l, width w, height h, and mass m are measured.
- The output variables volume V and density ρ are calculated from these input variables: $V = lwh,\ \rho = m/V$



Analysis of uncertainty in the ρ measurement involves 1) assessing the uncertainty in l, w, h, and m, and then 2) propagating the error to the calculated variables V and then ρ.

Error Propagation: If a functional relationship between input & output variables is available

Consider a relationship between an output variable y and a series of input variables x_j ($j = 1,2,3, \dots$) as:

$$y = f(x_1, x_2, x_3, \dots)$$

$$x_j = \underbrace{\bar{x}_j}_{} \pm \underbrace{\Delta x_j}_{\text{Uncertainty}}$$

The average value of y is obtained from the average values of x_j:

$$\bar{y} = f(\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots)$$

($\bar{x}_j$ is mean values)

Uncertainties or small changes in y (i.e., Δy_j) due to the change in x_j can be determined from a Taylor series as:

$$\Delta y_j = \frac{\partial y}{\partial x_j} \Delta x_j \quad (j = 1,2,3, \dots)$$

- Δx_j is uncertainty in the input variables.
- The derivative $\partial y / \partial x_j$ is called the **sensitivity** of y to the variable x_j (slope of the curve $y - x_j$).

Total Uncertainty

Total uncertainty in y resulting from all the uncertainties Δx_j $(j = 1,2,3, \dots)$:

Root Sum Square (RSS) Uncertainty:

$$\Delta y = \left[\left(\frac{\partial y}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\Delta x_2\right)^2 + \left(\frac{\partial y}{\partial x_3}\Delta x_3\right)^2 + \cdots\right]^{1/2}$$

Thus, the expected range is expressed as

$$y = \bar{y} \pm \Delta y$$

Common Expressions for RSS Uncertainty

Let a, b, c, ... be input variables and Δa, Δb, Δc, ... be the uncertainty in those variables.

Expression	RSS Uncertainty Relationship
$y = a \pm b \pm \cdots$	$\Delta y = [(\Delta a)^2 + (\Delta b)^2 + \cdots]^{1/2}$
$y = a \cdot b$	$\Delta y = [(b\Delta a)^2 + (a\Delta b)^2]^{1/2}$
$y = a \cdot b \cdot c$	$\Delta y = [(bc\Delta a)^2 + (ac\Delta b)^2 + (ab\Delta c)^2]^{1/2}$
$y = \frac{a}{b}$	$\Delta y = \left[\left(\frac{\Delta a}{b}\right)^2 + \left(\frac{a}{b^2}\Delta b\right)^2\right]^{1/2}$
$y = \frac{a \pm b}{e \pm f}$	$\Delta y = \left[\left(\frac{\Delta a}{e \pm f}\right)^2 + \left(\frac{\Delta b}{e \pm f}\right)^2 + \left(\frac{a \pm b}{(e \pm f)^2}\Delta e\right)^2 + \left(\frac{a \pm b}{(e \pm f)^2}\Delta f\right)^2\right]^{1/2}$
$y = \frac{a^{n_1} b^{n_2} c^{n_3} \cdots}{d^{m_1} e^{m_2} f^{m_3} \cdots}$	$\Delta y = y \cdot \left[\left(n_1 \frac{\Delta a}{a}\right)^2 + \left(n_2 \frac{\Delta b}{b}\right)^2 + \cdots + \left(m_1 \frac{\Delta d}{d}\right)^2 + \left(m_2 \frac{\Delta e}{e}\right)^2 + \cdots\right]^{1/2}$

$n_1, n_2, \ldots m_1, m_2 \ldots$ cab be integers (1,2,...)
or rational fractions (1/2, 3/4,...)

Example

Consider an electrical resistor with a voltage $V = 2.38 \pm 0.09$ volt across it. The resistor has a resistance of $R = 503 \pm 2\ \Omega$. Determine the power and the uncertainty of the power P dissipated by the resistor.

$$P = \frac{V^2}{R}$$

Example

If $y = \rho^3 U^{1/2} L^{2.3} / \mu^4$, find the RSS uncertainty in y, given ρ, U, L, μ and their uncertainties.

Uncertainty Tree

Uncertainty Tree represents a graphical depiction of the variable dependence in an error analysis. The output variable (whose uncertainty is ultimately desired) appears at the top of the tree, and the input variables (whose uncertainties are known) are listed at sublevels below it. Functional relationships (equations) are used to connect one level to another.

Example: Uncertainty Tree for **Kinetic Energy** of a homogeneous spherical object moving with a constant velocity by having the uncertainty in ρ, d, x, and t.

T_{KE}	is the kinetic energy of the object (J)
d	the object's diameter (m)
m	the object's mass (kg)
v	the velocity (m/s)
x	the distance traveled (m)
t	the time taken for the object to cross the distance x (s)
V	the object's volume (m^3)
ρ	the density (kg/m^3)

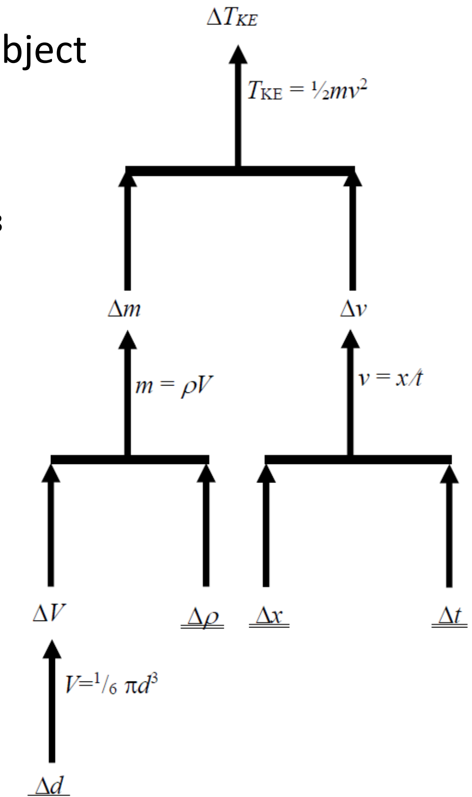
$$T_{KE} = \frac{1}{2} m v^2, \quad v = \frac{x}{t}, \quad m = \rho V, \quad V = \frac{1}{6}\pi d^3$$

①
$$\Delta V = \left[\left(\frac{\partial V}{\partial d}\Delta d\right)^2\right]^{\frac{1}{2}}$$

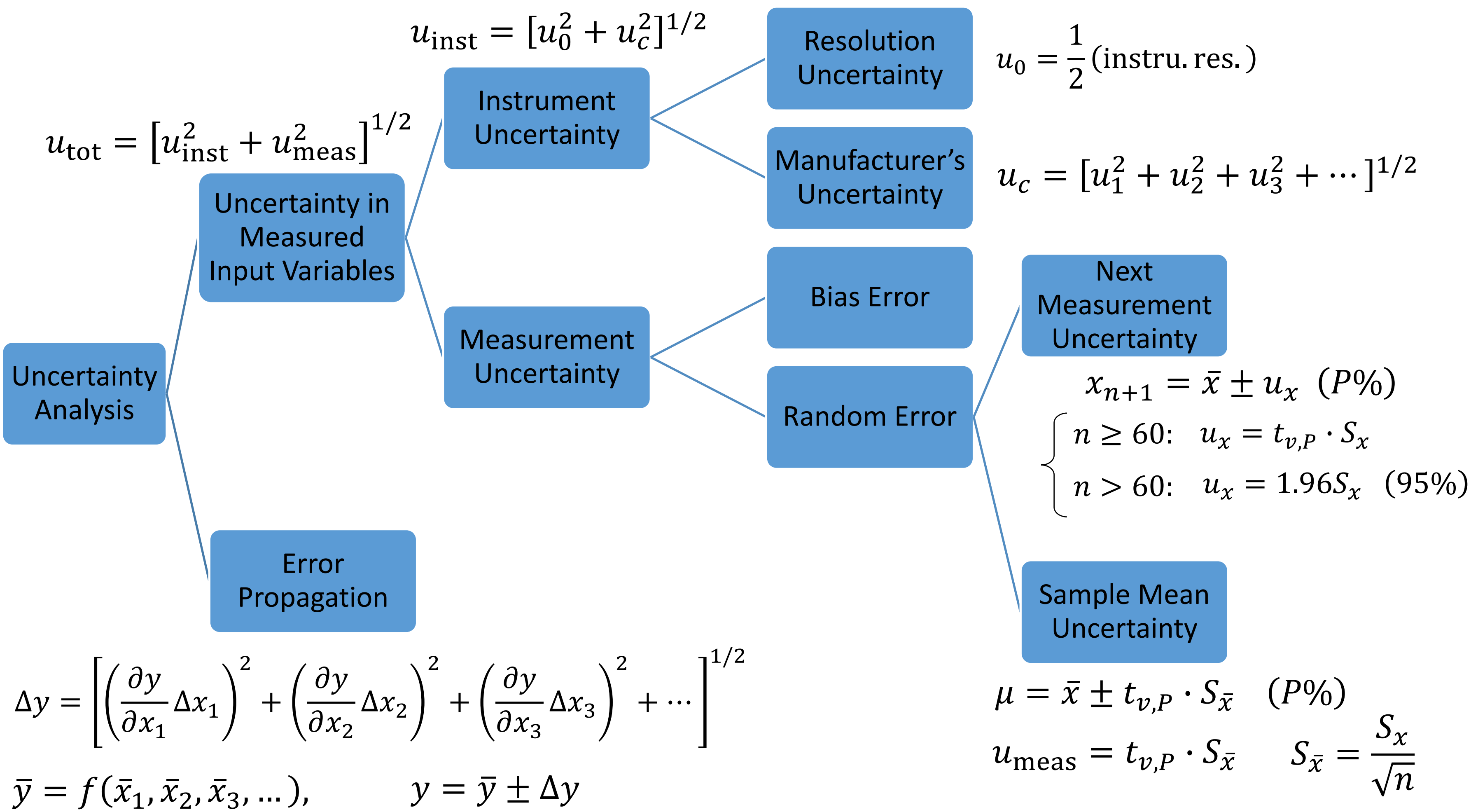
②
$$\Delta v = \left[\left(\frac{\partial v}{\partial x}\Delta x\right)^2 + \left(\frac{\partial v}{\partial t}\Delta t\right)^2\right]^{\frac{1}{2}}$$

$$\Delta m = \left[\left(\frac{\partial m}{\partial \rho}\Delta \rho\right)^2 + \left(\frac{\partial m}{\partial V}\Delta V\right)^2\right]^{\frac{1}{2}}$$

③
$$\Delta T_{KE} = \left[\left(\frac{\partial T_{KE}}{\partial m}\Delta m\right)^2 + \left(\frac{\partial T_{KE}}{\partial v}\Delta v\right)^2\right]^{\frac{1}{2}}$$



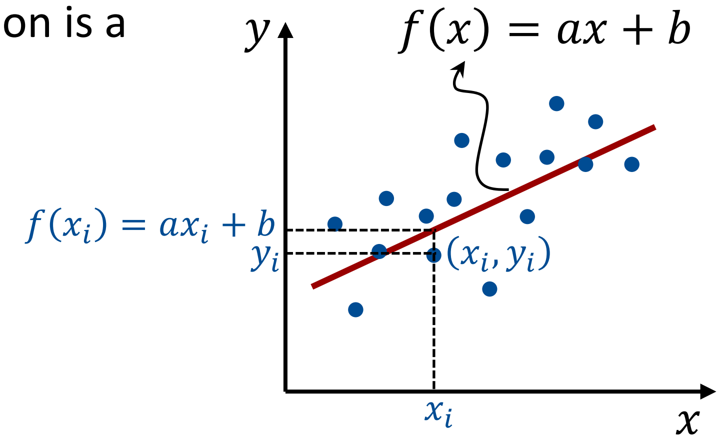
Summary


$u_{\text{inst}} = [u_0^2 + u_c^2]^{1/2}$
$u_{\text{tot}} = [u_{\text{inst}}^2 + u_{\text{meas}}^2]^{1/2}$
Resolution Uncertainty
$u_0 = \frac{1}{2}(\text{instru. res.})$
Instrument Uncertainty
Manufacturer's Uncertainty
$u_c = [u_1^2 + u_2^2 + u_3^2 + \cdots]^{1/2}$
Uncertainty in Measured Input Variables
Bias Error
Next Measurement Uncertainty
Uncertainty Analysis
Measurement Uncertainty
Random Error
$x_{n+1} = \bar{x} \pm u_x \quad (P\%)$
$n \geq 60: \quad u_x = t_{v,P} \cdot S_x$
$n > 60: \quad u_x = 1.96 S_x \quad (95\%)$
Error Propagation
Sample Mean Uncertainty
$\Delta y = \left[\left(\frac{\partial y}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial y}{\partial x_2}\Delta x_2\right)^2 + \left(\frac{\partial y}{\partial x_3}\Delta x_3\right)^2 + \cdots\right]^{1/2}$
$\bar{y} = f(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots), \qquad y = \bar{y} \pm \Delta y$
$\mu = \bar{x} \pm t_{v,P} \cdot S_{\bar{x}} \quad (P\%)$
$u_{\text{meas}} = t_{v,P} \cdot S_{\bar{x}} \qquad S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$

Linear Regression

Least-Squares Linear Regression

Regression Analysis is used to correlate experimental data by fitting mathematical functions. A common function is a straight line (**Linear Regression**).



n experimenta data point (x_i, y_i)

Fitting Function: $y = f(x) = ax + b$

Goal: Finding a and b in such a way that $f(x_i)$ and y_i are as close as possible.

Error: $e_i = f(x_i) - y_i$

Least-Squares method calculates the best-fitting line for the observed data by minimizing the sum of the squares of the vertical deviations from each data point to the line.

Squar Error: $e_i^2 = [f(x_i) - y_i]^2 = [ax_i + b - y_i]^2$

Least-Squares Linear Regression

Error for all data points:

$$E = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (ax_i + b - y_i)^2$$

New Goal: Finding a and b in such a way that E is minimized.

$$\frac{\partial E}{\partial a} = 0 \rightarrow \sum_{i=1}^{n} 2(ax_i + b - y_i)x_i = 0 \rightarrow \left(\sum_{i=1}^{n} x_i^2\right) a + \left(\sum_{i=1}^{n} x_i\right) b = \left(\sum_{i=1}^{n} x_i y_i\right)$$

$$\frac{\partial E}{\partial b} = 0 \rightarrow \sum_{i=1}^{n} 2(ax_i + b - y_i) = 0 \rightarrow \left(\sum_{i=1}^{n} x_i\right) a + nb = \left(\sum_{i=1}^{n} y_i\right)$$

$$a = \frac{n\sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$b = \frac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i y_i\right)}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}$$

Least-Squares Linear Regression

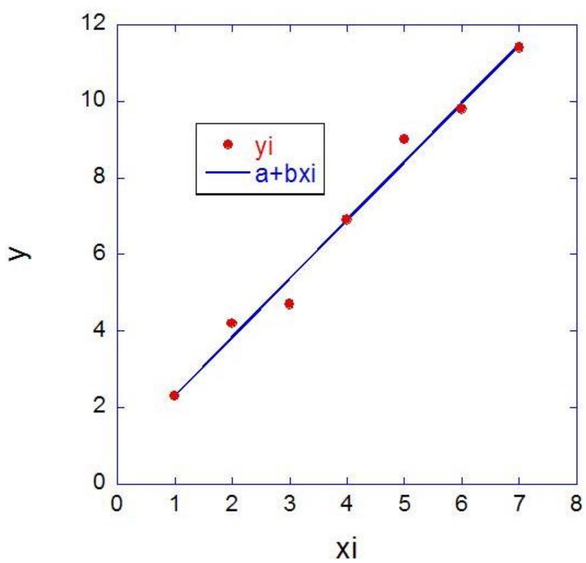
Uncertainty in the estimates of a and b:

$$u_a = \sigma_y \left(\frac{n}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \right)^{1/2}$$

$$u_b = \sigma_y \left(\frac{\sum_{i=1}^{n} x_i^2}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} \right)^{1/2}$$

where $\sigma_y = \left(\frac{E}{n-2} \right)^{1/2}$ Standard Deviation of y values from straight line

Example



i	xi	yi	xiyi	xixi	a+bxi	[yi-(a+bxi)]^2
1	1	2.3	2.3	1	2.314286	0.000204082
2	2	4.2	8.4	4	3.842857	0.12755102
3	3	4.7	14.1	9	5.371429	0.450816327
4	4	6.9	27.6	16	6.9	7.88861E-31
5	5	9	45	25	8.428571	0.326530612
6	6	9.8	58.8	36	9.957143	0.024693878
7	7	11.4	79.8	49	11.48571	0.007346939
k=	$\sum x_i$	$\sum y_i$	$\sum x_i y_i$	$\sum x_i^2$	E	=
7	28	48.3	236	140		0.937142857
	a=	0.785714				
	b=	1.528571				
	ua=	0.365893				
	ub=	0.081816				

This is just to show rough procedure. Significant Digits should be properly considered for actual report.