Ch2: Basic Concepts and Laws

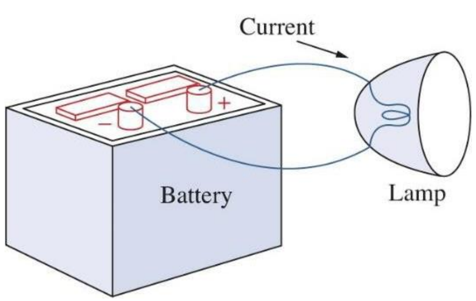
Introduction

Electric Circuit

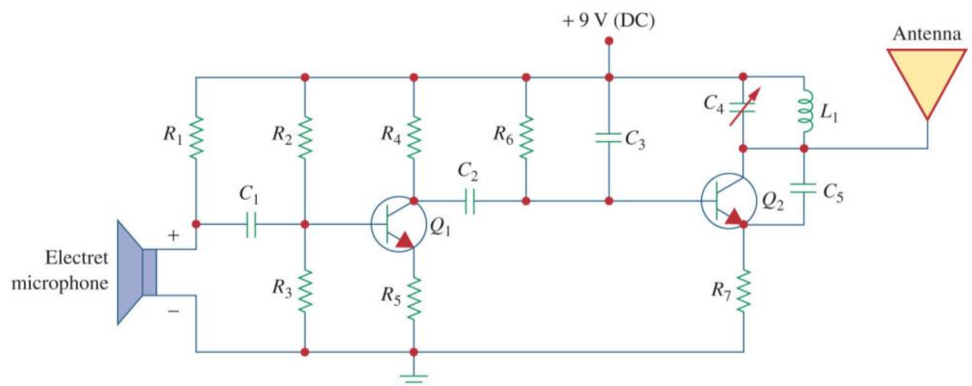
Electric Circuit Theory and **Electromagnetic Theory** are the two fundamental theories upon which all branches of electrical engineering are built.

A basis for many EE course such as
Power, Electric Machines, Control, Electronics, Communications, and Instrumentation

- An **Electric Circuit** is an interconnection of electrical elements.
- **Circuit Analysis** is the process of determining voltages across (or the currents through) the elements of the circuit.



A simple electric circuit consists of three basic elements: a battery, a lamp, and connecting wires.



A more complicated electric circuit which is a radio transmitter.

Electric Circuit

- **Theory**: You will learn various *analysis* methods in lectures to analyze the behavior of such electric circuits.
  - How does the circuit respond to a given input?
  - How do the elements and devices in the circuit interact?
- **Practice**: You will also learn how to *build and test* basic electric circuits through labs and projects.

Topics Covered in This Course (Tentative):

- Intro to circuits: currents, voltages; power/energy; circuit elements
- DC Circuits
  - Basic Circuit Laws (Ohm, Kirchhoff)
  - Circuit Analysis: nodal analysis and mesh analysis
  - Circuit Theorems: Thevenin, Norton, Superposition
  - Operational Amplifiers: ideal, inverting/non-inverting, summing and difference
  - Capacitors and Inductors
  - Diodes and Transistors
- AC Circuits
  - Sinusoids and Phasors
  - AC Power Analysis
  - Three-phase Circuits

System of Units

- When taking measurements, we must use units to quantify values.
- We use the **International Systems of Units** (SI) made up of seven **Base Units** and many **Derived Units**.
  - **Base Units** form the core building blocks of any unit system, and they are independent of one another.
  - **Derived Units** are combinations of several base units.

Quantity	SI Base Unit	Abbr.
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	Kelvin	K
Amount of substance	mole	mol
Light intensity	candela	cd

Standard Prefixes in the SI

Base and derived units in the **SI** are often combined with a prefix which is a **power-of-ten exponent** to shorten the representation of a numerical value and to reduce calculations.

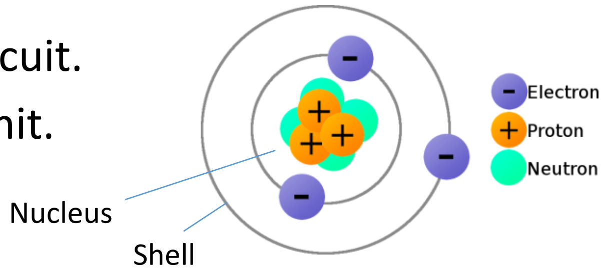
7,000,000 W (watt) $\Rightarrow$ 7 MW (megawatt)

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Electric Charge, Current, and Voltage

Electric Charge

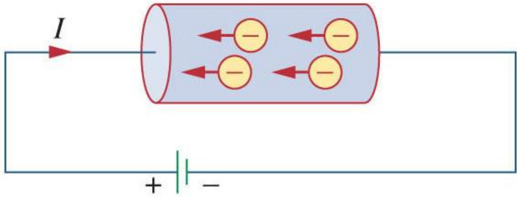
- **Electric Charge** is the most basic quantity in an electric circuit.
- Charge is measured in **Coulombs (C)**, which is a derived unit.



- All matter is made of fundamental building blocks known as **atoms** and each atom consists of electrons, protons, and neutrons. The charge e on an electron is negative and $e = -1.602 \times 10^{-19}$ C, while a proton carries a positive charge of the same magnitude as the electron. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.
- One Coulomb is quite large. In 1 C of charge, there are $1/(1.602 \times 10^{-19}) = 6.24 \times 10^{18}$ electrons. In lab, it is in the order of pC, nC, or μC.
- According to experimental observations, electric charge exists in discrete quantities, integral multiples of the electron charge -1.602×10^{-19} C.
- The law of conservation of charge states that charge can neither be created nor destroyed, only transferred. Thus, the algebraic sum of the electric charges in a system does not change.

Electric Current

When a conducting wire is connected to a battery, the charges are forced to move; positive charges move in one direction while negative charges move in the opposite direction. This motion of charges creates **electric current**.



Electric Current is the time rate of change of charge, measured in amperes (A).

$$i = \frac{dq}{dt}$$

i: current (A), q: charge (C), t: time (s)

1 A = 1 C/s

The charge transferred between time t_0 and t is obtained by integration:

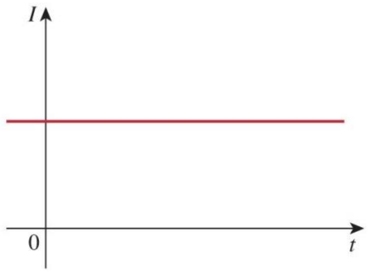
$$Q = \int_{t_0}^{t} i dt$$

❖ By a universally accepted convention, current is the flow of positive charges. That is, opposite to the actual flow of negative charges. We use this convention in this course.

DC and AC

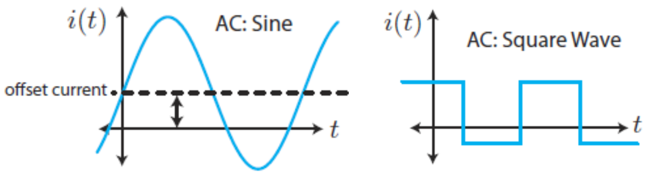
A **Direct Current (DC)** is a current that flows only in one direction and can be constant or time varying.

Example: The current coming from a battery.

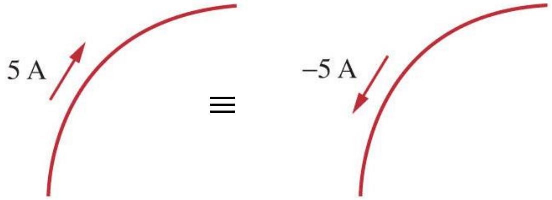


An **Alternating Current (AC)** is a current that changes direction (flows in both directions) and varies with respect to time.

Example: The current coming from your home outlets.



- By convention, we use the symbol I to represent a constant DC current. If the current varies with respect to time (either DC or AC), we use the symbol i or $i(t)$.
- A positive current through a component is the same as a negative current flowing in the opposite direction.



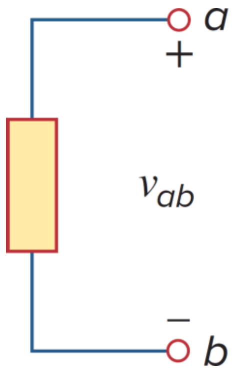
Examples

The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

Determine the total charge entering a terminal between $t = 1$ s and $t = 2$ s if the current passing the terminal is $i = (3t^2 - t)$ A.

Voltage

Voltage (or **Potential Difference**) v_{ab} between two points a and b in an electric circuit is the energy required to move a unit (+) charge from a reference point b or (−) to another point a or (+), measured in volts (V).



$$v_{ab} = \frac{dw}{dq}$$

w: energy in joules (J), q: charge in coulombs (C)
1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

- Voltage is a relative quantity. An absolute voltage usually is implicitly referenced to a known point in the circuit (ground or zero voltage).

$$v_{ab} \equiv v_a - v_b$$

- It is common to use the ground symbol to simplify electrical circuits. All voltages are implicitly referenced to the ground terminal.

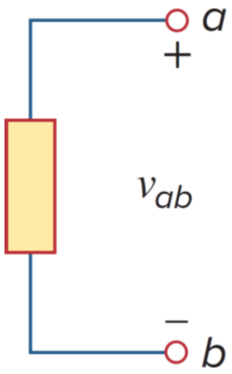
Voltage

- The v_{ab} can be interpreted in two ways:

(1) Point a is at a potential of v_{ab} volts higher than point b, or
(2) the potential at point a with respect to point b is v_{ab}.

$$v_{ab} = -v_{ba}$$

- In electrical circuits, the path of motion is well defined by wires or circuit elements. We usually label the terminals of a component as positive and negative to denote the voltage drop across the component.



- Like electric current, a constant voltage is called a DC voltage and is represented by V, whereas a time-varying voltage is called an AC voltage and is represented by $v(t)$. A DC voltage is commonly produced by a battery; AC voltage is produced by an electric generator.

Example

An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Power and Energy

Power

Power p is the time rate of expending or absorbing energy, measured in watts (W).

$$p = \frac{dw}{dt}$$

w: energy in joules (J), t: time in seconds (s)
W=J/s

$$p = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi \quad \rightarrow \quad p = vi$$

- The power absorbed or supplied by an element is the product of the voltage across the element and the current through it.
- The power p is a time-varying quantity and is called the **instantaneous power**.
- The energy absorbed or supplied by an element from time t_0 to time t is

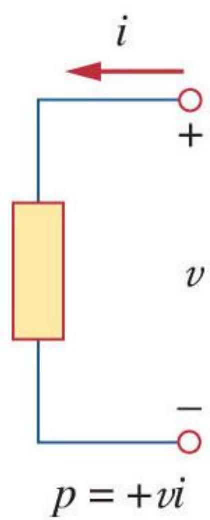
$$w = \int_{t_0}^{t} p dt = \int_{t_0}^{t} vi dt$$

Energy is measured in joules (J).

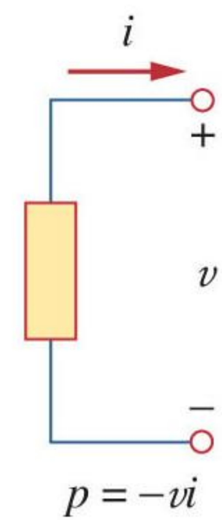
Passive Sign Convention

Power can be positive or negative. Current direction and voltage polarity play a major role in determining the sign of power.

Passive Sign Convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.



- We will follow the passive sign convention in this course.



If the power has a + sign, power is being delivered to or absorbed by the element.

If power has a − sign, power is being supplied by the element (such as a battery).

Conservation of Energy

Based on The **law of conservation of energy**, the algebraic sum of power in a circuit, at any instant of time, must be zero (the total power supplied to the circuit must balance the total power absorbed).

$$\sum p = 0$$

Examples

Find the power delivered to an element at $t = 3$ ms if the current entering its positive terminal is $i = 5\cos 60\pi t$ A and the voltage is $v = 3i$.

How much energy does a 100-W electric bulb consume in two hours?

Circuit Elements

Circuit Elements

An electric circuit is an interconnection of the elements. 5 ideal basic circuit elements:

- voltage source
- current source

} **active elements**, capable of generating electric energy to the circuit.

- resistor
- inductor
- capacitor

} **passive elements**, incapable of generating electric energy to the circuit.

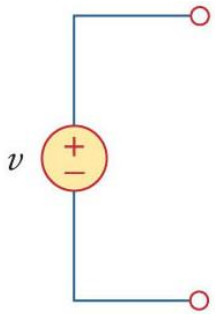
There are two types of voltage and current sources:

independent sources and **dependent sources**

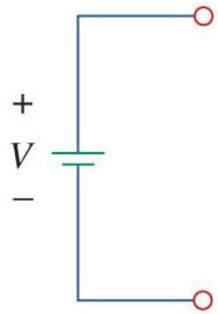
Ideal Independent Voltage & Current Sources

An **ideal independent source** provides a specified voltage/current completely independent of other circuit elements.

An **ideal independent voltage source** delivers to the circuit whatever current is necessary to maintain its designated voltage.

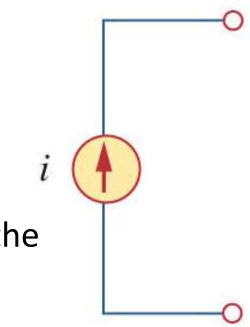


Symbol for constant or time-varying voltage



Symbol for constant voltage (DC).

An **ideal independent current source** delivers to the circuit whatever voltage is necessary to maintain its designated current.

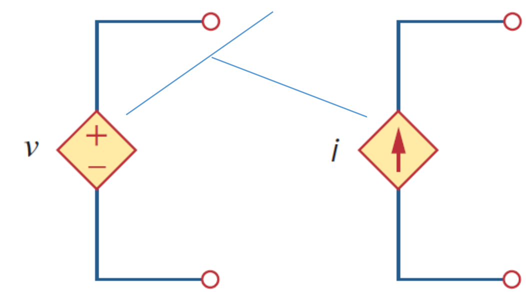


The arrow indicates the direction of current.

Ideal Dependent Voltage & Current Sources

An **ideal dependent (or controlled) source** is an active element in which the source quantity is controlled by another voltage or current.

Diamond shape



dependent voltage source

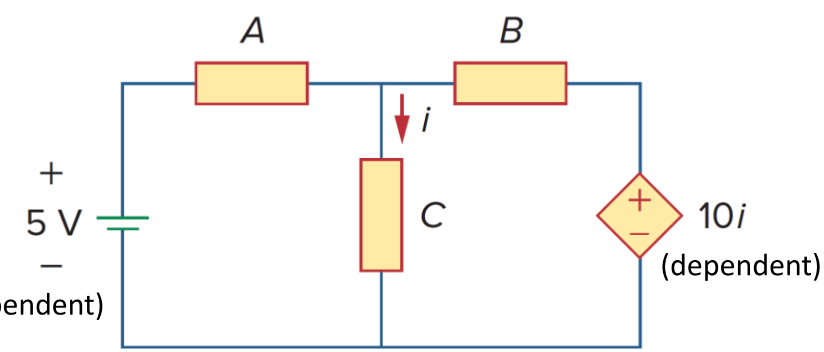
dependent current source

Four possible types of dependent sources:

1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

Example:

In this circuit the voltage $10i$ of the voltage source depends on the current i through element C.



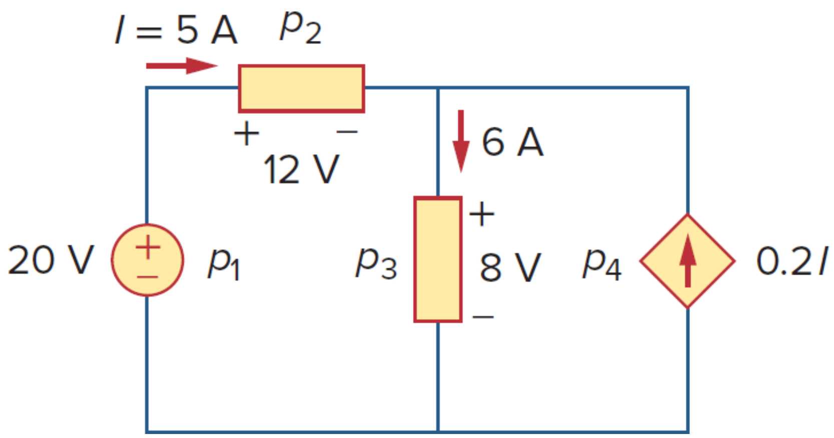
Remarks

- An ideal voltage source (dependent or independent) will produce any current required to ensure that the terminal voltage is as stated, whereas an ideal current source will produce the necessary voltage to ensure the stated current flow. Thus, **an ideal source could in theory supply an infinite amount of energy**. However, in reality, voltage and current source sources do have upper voltage and current limits.

- Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits.

Example

Calculate the power supplied or absorbed by each element in the figure based on power sign convention.

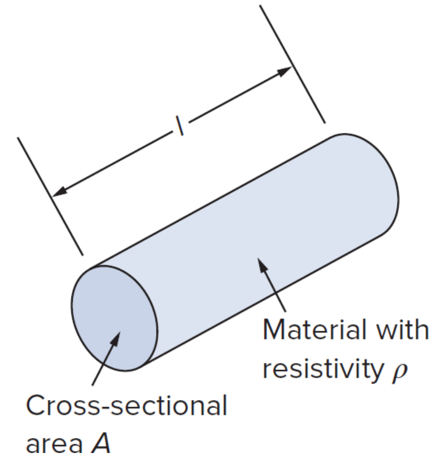


Ohm’s Law

Resistivity

- Materials tend to resist the flow of electricity (current) through them. This property is called **Resistance** and is represented by the symbol R.
- The resistance of an object is a function of its length l, and cross-sectional area A, and the material's resistivity ρ (in ohm-meters):

$$R = \rho \frac{l}{A}$$



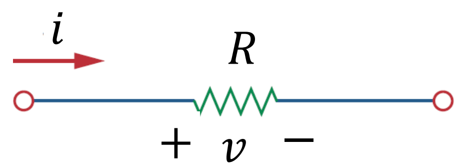
Resistivities of common materials.

Material	Resistivity (Ω·m)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon	4×10^{-5}	Semiconductor
Germanium	47×10^{-2}	Semiconductor
Silicon	6.4×10^{2}	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Good conductors, such as copper and aluminum, have low resistivities, while insulators, such as mica and paper, have high resistivities.

Ohm's Law

The circuit element used to model the current-resisting behavior of a material is the **Resistor**, which is the simplest passive element.



Georg Ohm (1789-1854)

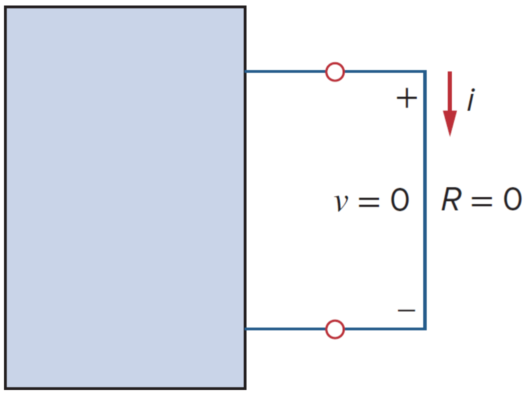
Ohm's Law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v \propto i \quad \longrightarrow \quad v = iR$$

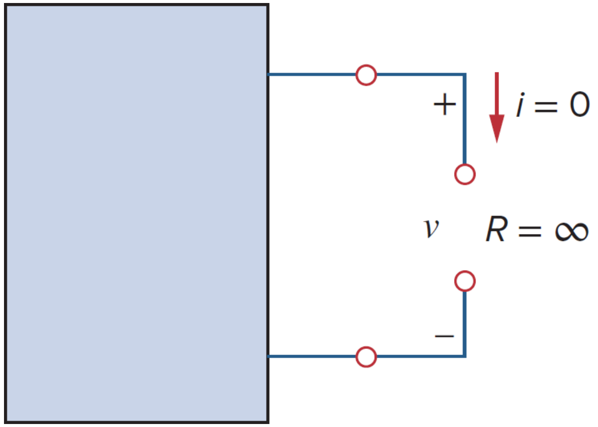
- The resistance of an element is measured in units of ohms (Ω): 1 Ω = 1 V/A
- The higher the resistance, the less current will flow through for a given voltage.
- The direction of current i and the polarity of voltage v must conform with the passive sign convention. If current flows from a higher potential to a lower potential, $v = iR$. If current flows from a lower potential to a higher potential, $v = -iR$.

Short and Open Circuits

- A connection with almost zero resistance ($R = 0$) is called a **Short Circuit**.
- All points on the wire are at the same potential.
- Ideally, any current (determined by the circuit) may flow through the short.
- In practice, this is a connecting wire.



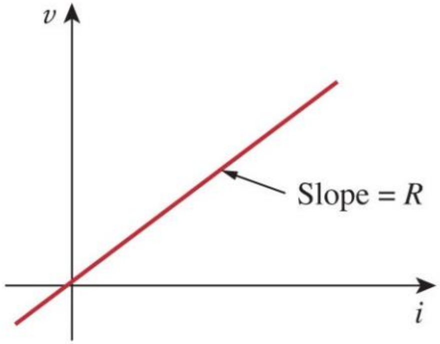
- A connection with infinite resistance ($R = \infty$) is called an **Open Circuit**.
- Here no matter the voltage, no current flows.
- Voltage difference can exist, as determined by the circuit.



Linearity

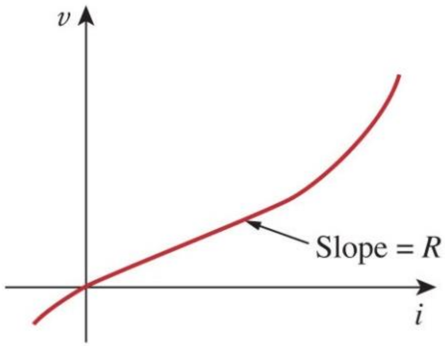
- Not all materials obey Ohm's Law.

- Resistors that obeys Ohm's law are called **linear resistors** because their current voltage relationship is always linearly proportional.



(a)

- A **nonlinear resistor** does not obey Ohm's law. Its resistance varies with current. Diodes and light bulbs are examples of non-linear elements.

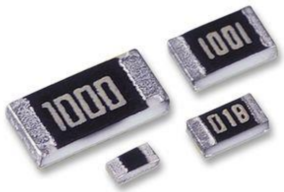


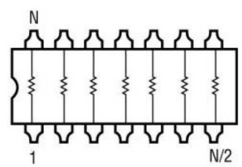
(b)

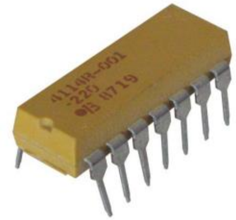
Note: Although all practical resistors may exhibit nonlinear behavior under certain conditions, we will assume in this course that all resistors are linear.

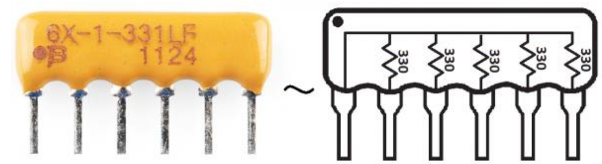
Resistor Packages

Axial-lead, Surface Mount, Dual In-line Package (DIP), Single In-line Package (SIP)



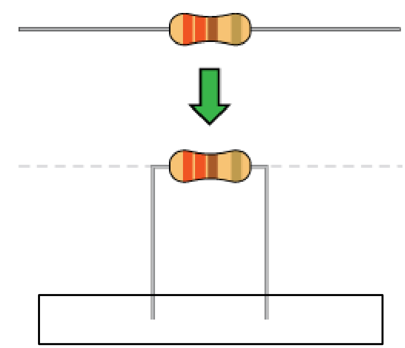






(multiple resistors in a package)

Axial-Lead Resistors come with long pliable leads which can be stuck into a breadboard. These resistors usually use the color-band system to display their value.



- The power rating of these resistors are usually somewhere between 1/8W (0.125W) and 1W. The most common resistors you will use in ordinary electronic circuitry are 1/4-watt, 5% tolerance carbon or metal-film resistors.

Reading Resistance of Axial-Lead Resistors

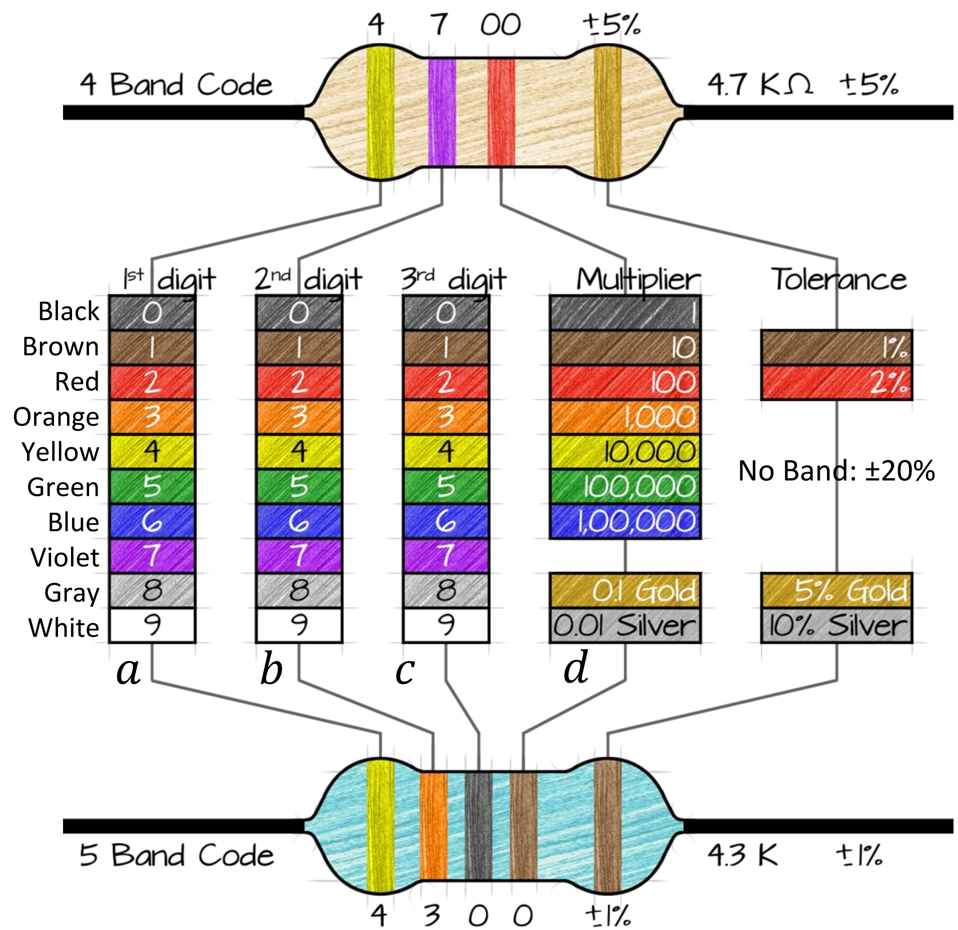
4-Band Code:
The **first two** bands indicate the two most-significant digits of the resistor's value. The **third** band is a weight value, which multiplies the two significant digits by a power of ten.

The **last**, tolerance band is often clearly **separated** from the value bands.

$$R = ab \times d \pm \text{tolerance } (\%)$$

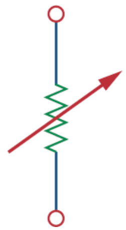
5-Band Code:
It is similar to explanation of 4-Band Code.

$$R = abc \times d \pm \text{tolerance } (\%)$$

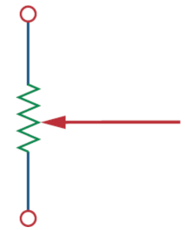


Variable Resistors

Variable Resistors are available that provide **a range of resistance values** controlled by a mechanical screw, knob, or linear slide. The most common type is called a **potentiometer**, or **pot**.

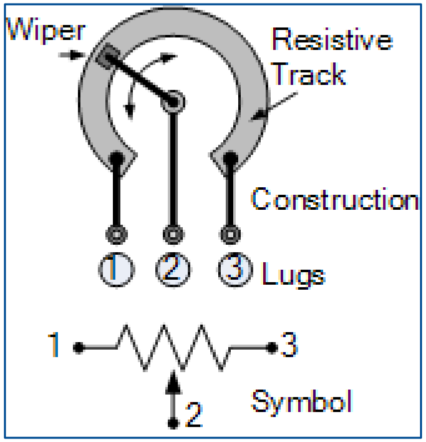


variable resistor symbol



potentiometer symbol

The pot is a three-terminal element with a sliding contact or wiper. By sliding the wiper, the resistances between the wiper terminal and the fixed terminals vary.



Conductance

Conductance G is the reciprocal of resistance R, which is a measure of how well an element will conduct electric current.

$$G = \frac{1}{R} = \frac{i}{v}$$

The unit of conductance is the mho (ohm spelled backward) with symbol ℧, the inverted omega, or siemens (S) in SI.

1 S = 1 ℧ = 1 A / V

Power Dissipation

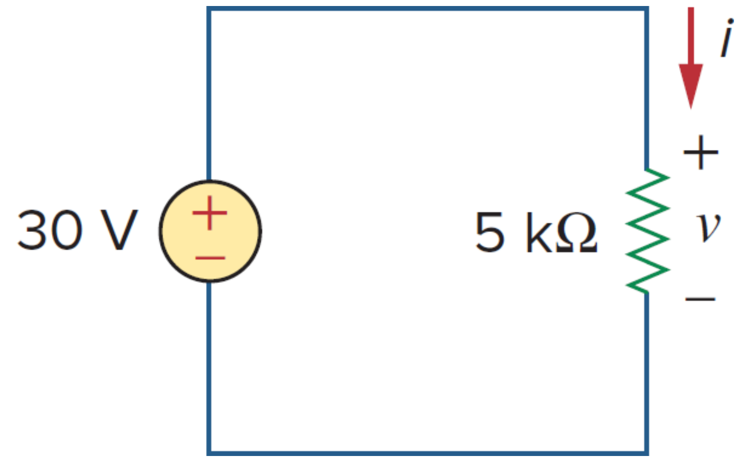
- Running current through a resistor dissipates power.

$$\begin{aligned} p &= vi \\ v &= Ri \end{aligned} \quad \Rightarrow \quad p = vi = i^2R = \frac{v^2}{R}$$

- The power dissipated in a resistor is a non-linear function of current or voltage.
- Power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit. This confirms the idea that a resistor is a passive element, incapable of generating energy.

Example

In the circuit shown, calculate the current i, the conductance G, and the power p.



Kirchhoff's Laws

Branch, Node, and Loop

- Circuit elements can be interconnected in multiple ways. To understand this, we need to be familiar with some network topology concepts, such as **branch**, **node**, and **loop**.

- A **branch** represents a single (two-terminal) element such as a voltage source or a resistor.

- A **node** is the point of connection between two or more branches. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node.

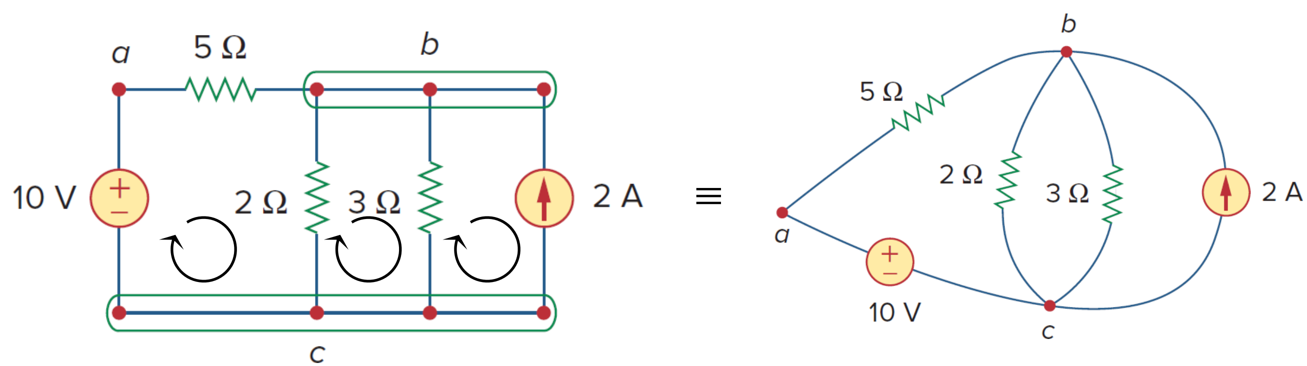
- A **loop** is any closed path in a circuit. A loop is **independent** if it contains at least one branch which is not a part of any other independent loop.

Example:

5 branches

3 nodes a, b, and c

3 independent loops

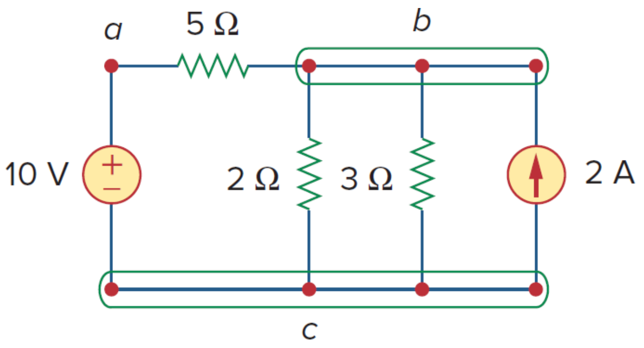


Elements in Series and Parallel

- Two or more elements are in **series** if they share a single node and consequently carry the same current.
- Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.
- Elements may be connected in a way that they are **neither in series nor in parallel**.

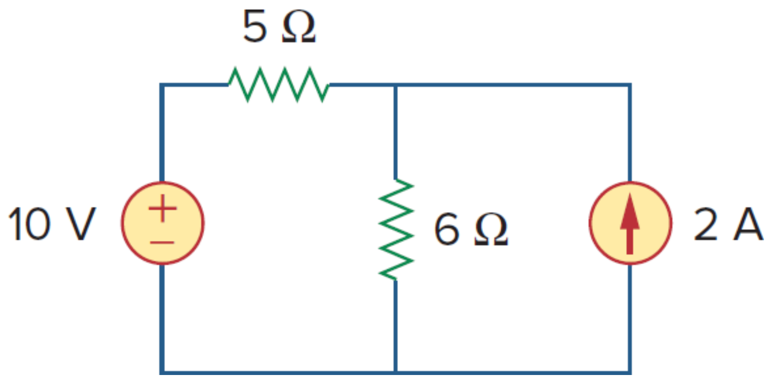
Example:

- The 10-V voltage source and the 5-Ω resistor are in series.
- The 2-Ω resistor, the 3-Ω resistor, and the 2-A current source are in parallel.
- The 5-Ω and 2-Ω resistors are neither in series nor in parallel with each other.



Example

Determine the number of branches and nodes in the circuit shown in figure. Identify which elements are in series and which are in parallel.



Kirchoff's Laws

Ohm's law by itself is not sufficient for circuit analysis.
Kirchoff's Laws complete the needed tools.

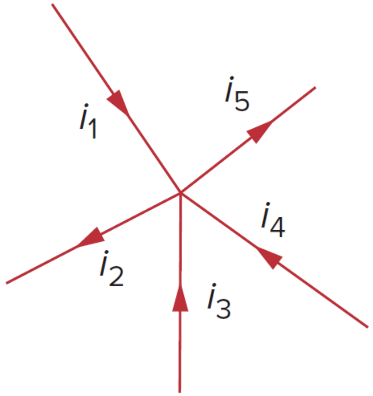
- There are two Kirchhoff's laws:
  - Kirchhoff's Current Law (KCL)
  - Kirchhoff's Voltage Law (KVL)

Gustav Robert Kirchhoff
(1824–1887)

Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero. This is based on conservation of charge.

$$\sum_{n=1}^{N} i_n = 0$$

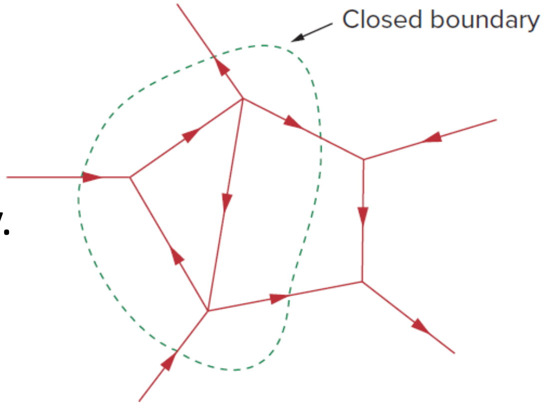


N is the number of branches connected to the node,
i_n is the nth current entering (or leaving) the node.
Currents entering a node are positive,
Currents leaving the node are negative.

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$
$$\Downarrow$$
$$i_1 + i_3 + i_4 = i_2 + i_5$$

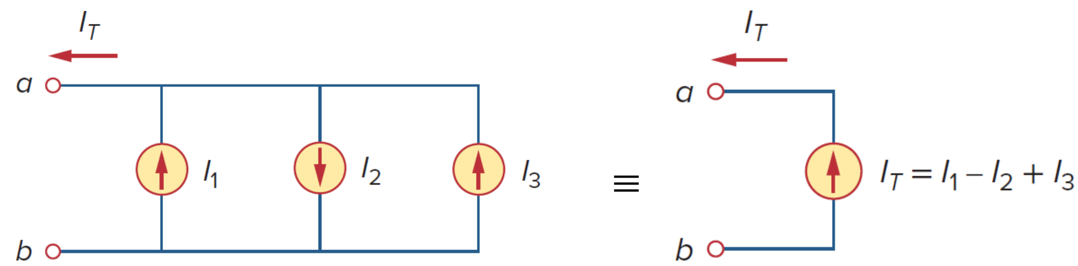
An alternative form of KCL:
The sum of the currents entering a node is equal to the sum of the currents leaving the node.

Generalization of KCL: KCL can be applied to any closed boundary.

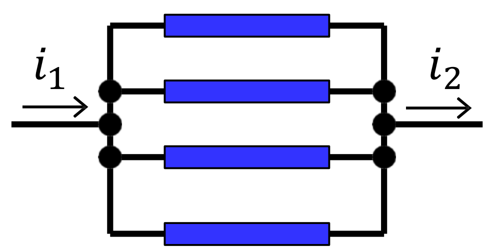


Kirchhoff's Voltage Law (KCL) (cont.)

Example: Combining current sources in parallel.



Generalized KCL Example:

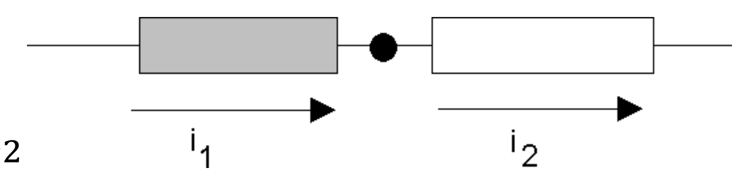


$$i_1 = i_2$$

A Major Implication of KCL:

All of the elements that are connected in series carry the same current.

$$i_1 = i_2$$

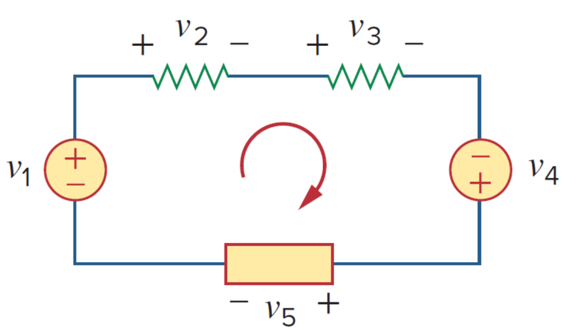


Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero. This is based on conservation of energy.

$$\sum_{m=1}^{M} v_m = 0$$

M is the number of voltages in the loop
v_m is the mth voltage



- Assume a current direction on each loop of the circuit.
- The voltage drops across each passive element in the direction of the loop current.
- The polarity of voltage across a voltage source and the direction of current through a current source must always be maintained as given. The voltage rises (from - to +) or drops (from + to -) across an element (like voltage source).

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0 \longrightarrow v_2 + v_3 + v_5 = v_1 + v_4$$

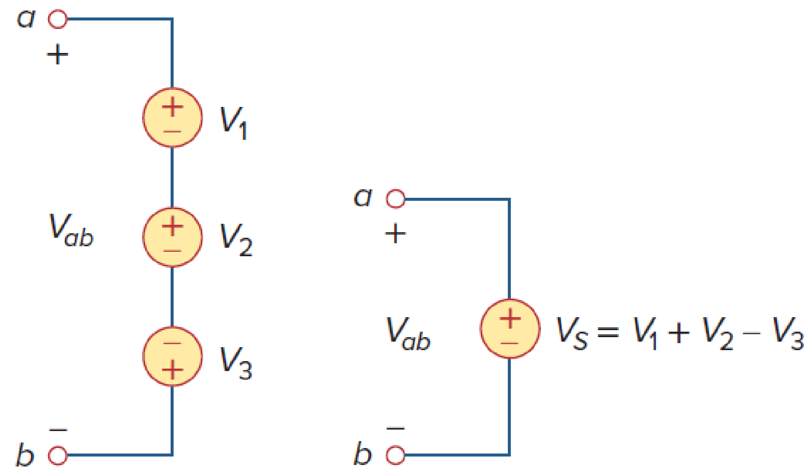
An alternative form of KVL: Sum of voltage drops = Sum of voltage rises

Kirchhoff's Voltage Law (KVL) (cont.)

Example: Combining voltage sources in series.

$$-V_{ab} + V_1 + V_2 - V_3 = 0$$

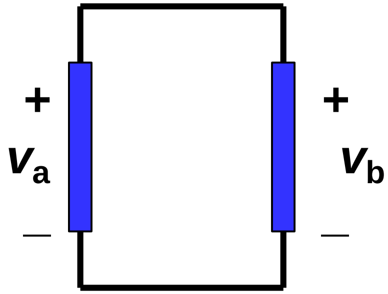
$$V_{ab} = V_1 + V_2 - V_3$$



A Major Implication of KVL:

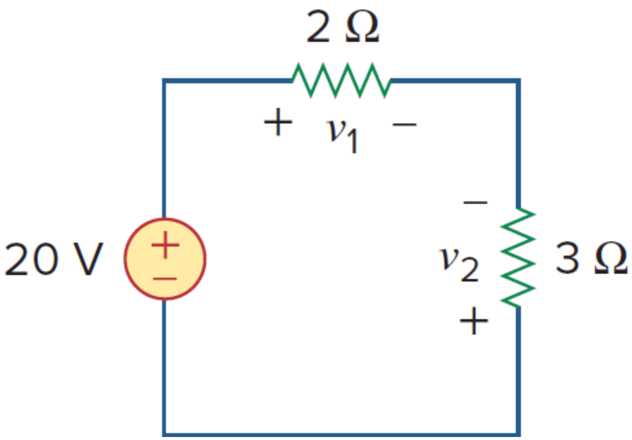
All of the elements that are connected in parallel carry the same voltage.

$$v_b - v_a = 0 \longrightarrow v_b = v_a$$



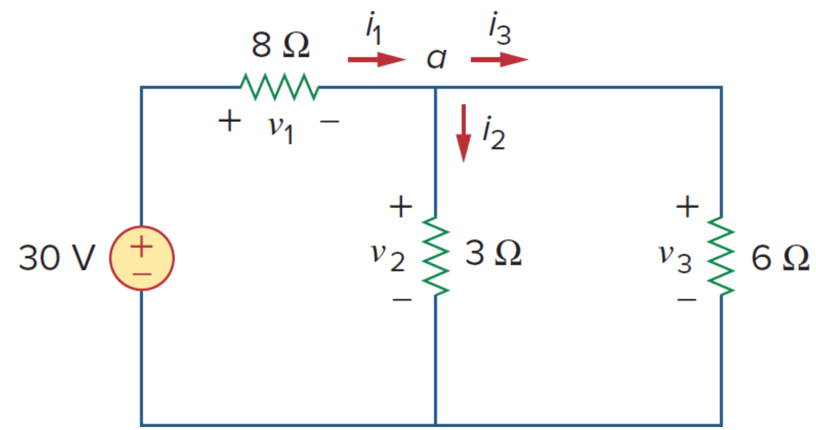
Example

For the circuit, find voltages v_1 and v_2.



Example

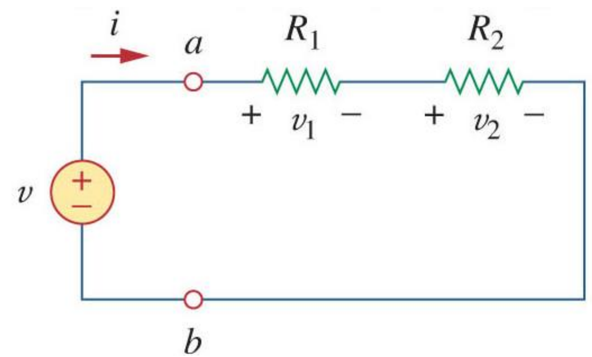
Find currents and voltages in the circuit.



Series/Parallel Resistors

Series Resistors

Two resistors are considered in series if the same current pass through them.



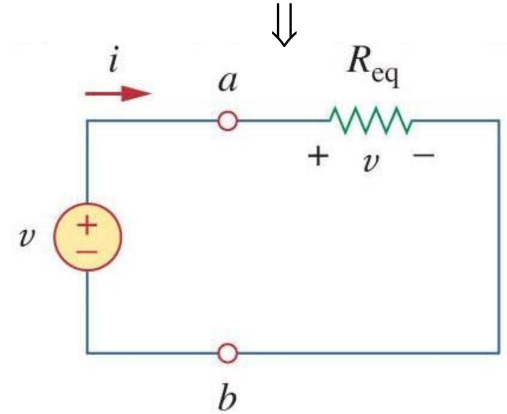
Applying Ohm's law to both resistors:

$$v_1 = R_1 i$$

$$v_2 = R_2 i$$

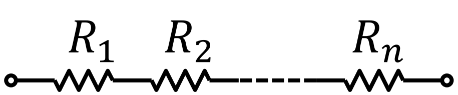
Applying KVL: $v - v_1 - v_2 = 0$

Combining equations: $v = v_1 + v_2 = i\underbrace{(R_1 + R_2)}_{R_{\text{eq}}}$



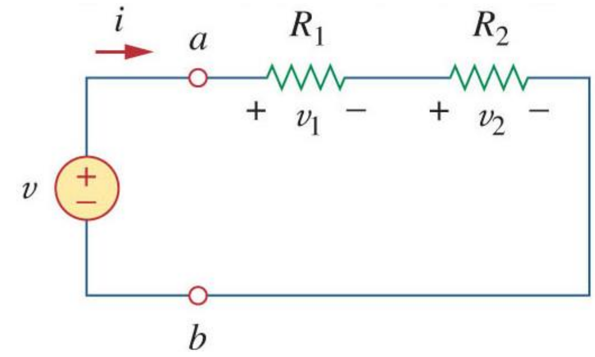
The total resistance of resistors connected in **series** is the sum of their individual resistance values.

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n = \sum_{i=1}^{n} R_i$$



Voltage Division

We can find the voltage drop across any one resistor.



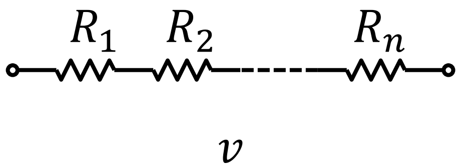
The current through all the resistors is the same, so using Ohm's law:

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

- Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the **principle of voltage division**, and its circuit is called a **voltage divider**.

❖ In general, if a voltage divider has n resistors in series with the source voltage v, the ith resistor (R_i) will have a voltage drop of

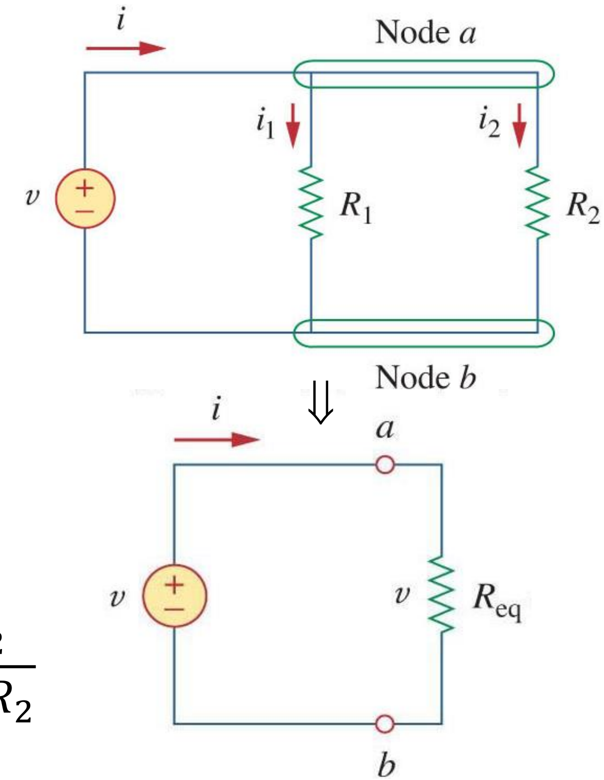
$$v_i = \frac{R_i}{R_1 + R_2 + \cdots + R_n} v$$



Parallel Resistors

When resistors are in parallel, the voltage drop across them is the same.

$$v = i_1 R_1 = i_2 R_2$$



Applying KCL: $i = i_1 + i_2$

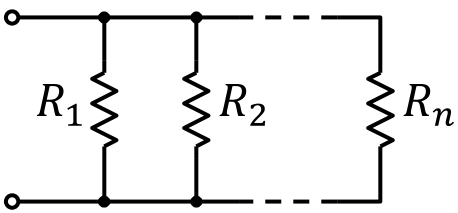
Combining equations: $i = \overbrace{(1/R_1 + 1/R_2)}^{1/R_{eq}} v$

The equivalent resistance is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

The total resistance of resistors connected in **parallel** is the reciprocal of the sum of the reciprocals of the individual resistors.

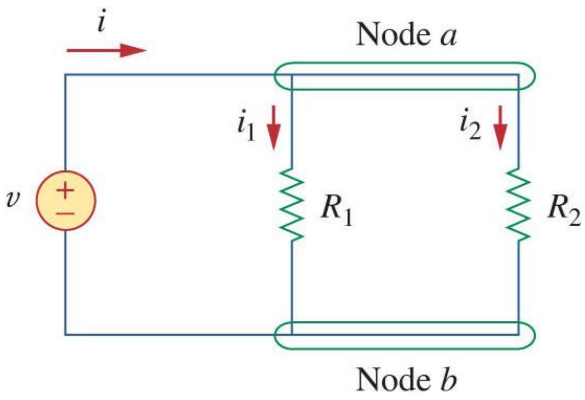
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} = \sum_{i=1}^{n} \frac{1}{R_i}$$



Note: If $R_1 = \cdots = R_N = R$, then $R_{eq} = R/N$.

Current Division

We can find the current i_1 and i_2. Given the current entering the node, the voltage drop across the equivalent resistance will be the same as that for the individual resistors.



$$v = iR_{\text{eq}} = \frac{iR_1R_2}{R_1 + R_2}$$

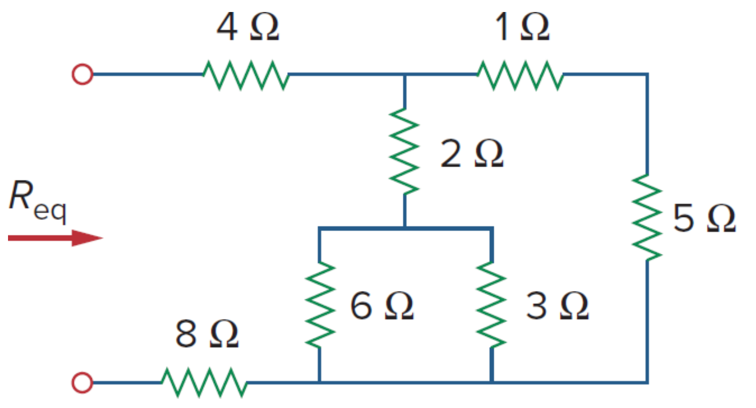
Combining with Ohm's law to get the current through each resistor:

$$i_1 = \frac{iR_2}{R_1 + R_2} \qquad i_2 = \frac{iR_1}{R_1 + R_2}$$

- Notice that the total current i is shared by the resistors in inverse proportion to their resistances; the larger current flows through the smaller resistance. This is called the **principle of current division**, and its circuit is called a **current divider**.

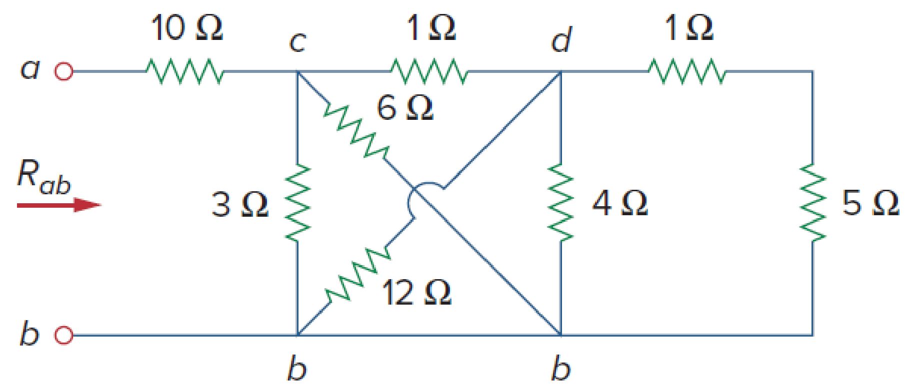
Example

Find R_{eq} for the circuit.



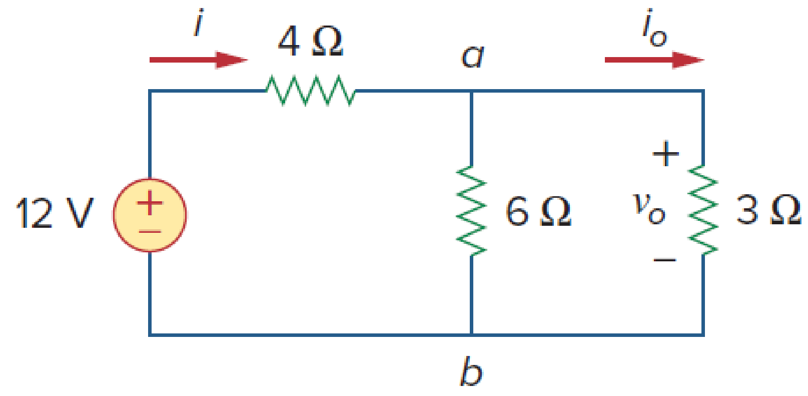
Example

Find R_{eq} for the circuit.



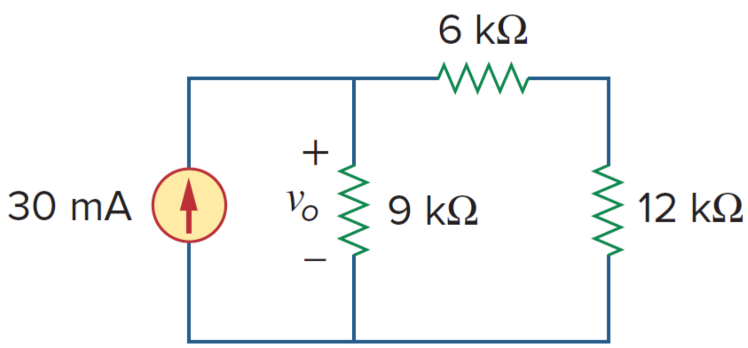
Example

Find i_o and v_o in the circuit. Calculate the power dissipated in the 3-Ω resistor.



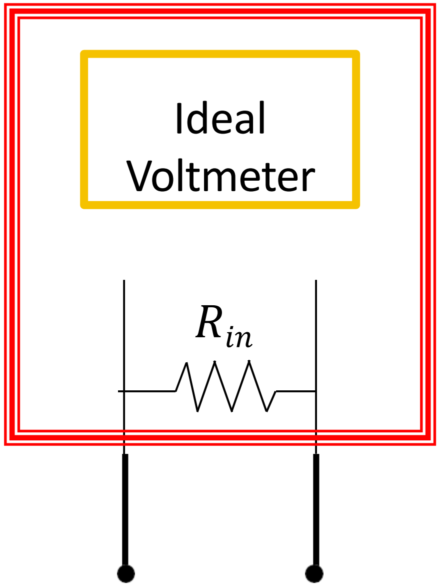
Example

For the circuit shown, determine: (a) the voltage v_o, (b) the power supplied by the current source, (c) the power absorbed by each resistor.



Measuring Voltage (Voltmeter)

- To measure the voltage drop across an element in a real circuit, insert a voltmeter (digital multimeter in voltage mode) **in parallel** with the element.
- Voltmeters are characterized by their "voltmeter input resistance" (R_{in}). Ideally, this should be very high (typical value 10 MΩ)



Measuring Current (Ammeter)

- To measure the current flowing through an element in a real circuit, insert an ammeter (digital multimeter in current mode) **in series** with the element.
- Ammeters are characterized by their "ammeter input resistance" (R_{in}). Ideally, this should be very low (typical value 1Ω).