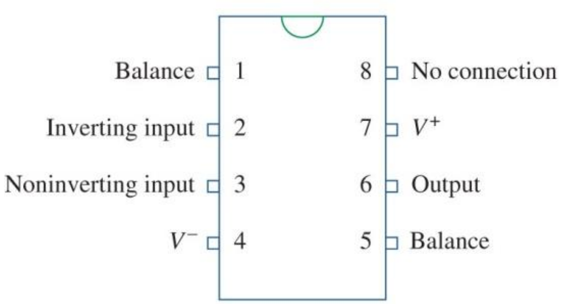
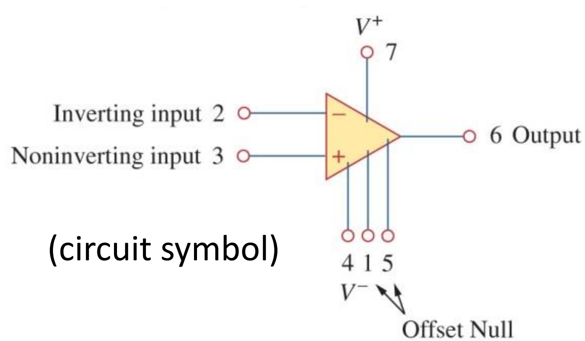
Ch6: Operational Amplifiers

Operational Amplifiers

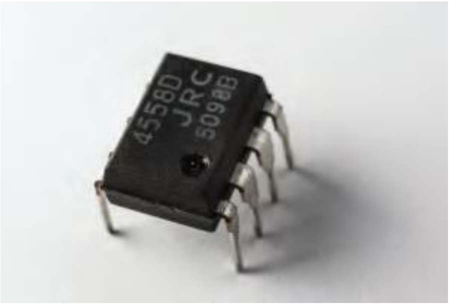
Operational Amplifier

- An **Operational Amplifier**, or **Op Amp** for short, is an electronic unit that behaves like a voltage-controlled voltage source.
- It is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration when external components, such as resistors and capacitors, are connected to its terminals.
- Op amps are commercially available in integrated circuit (IC) packages in several forms. A typical one is the eight-pin dual in-line package (or DIP) as shown.
- Pin 8 is unused, and Pins 1 and 5 are of little concern to us. The five important terminals are: inverting input (Pin 2), noninverting input (Pin 3), output (Pin 6), positive and negative power supplies (Pins 7, 4).





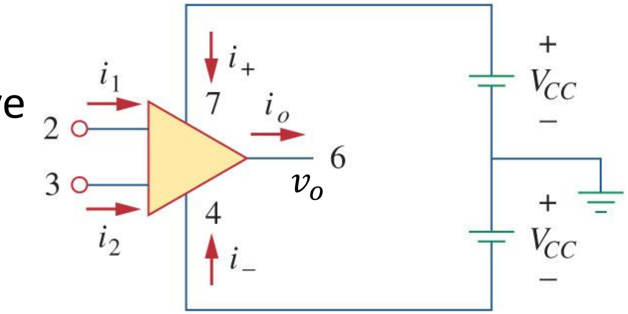
(circuit symbol)



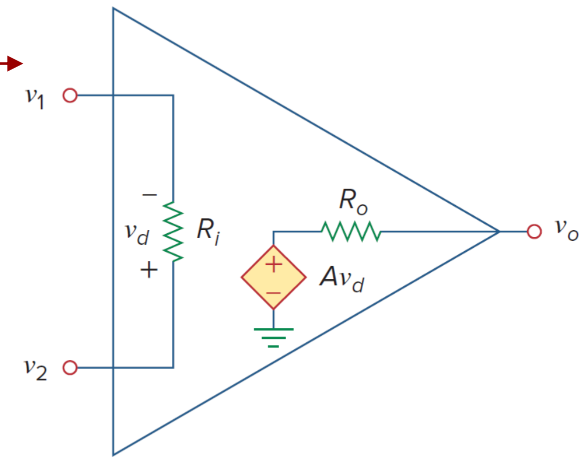
Operational Amplifier

- Most op-amps use two voltage sources, with a ground reference between them. This gives a positive and negative supply voltage.

$$i_o = i_1 + i_2 + i_+ + i_-$$
$$-V_{CC} \leq v_o \leq V_{CC}$$



- The equivalent circuit model of an op amp is shown. The output section consists of a voltage-controlled source in series with the output resistance R_o.

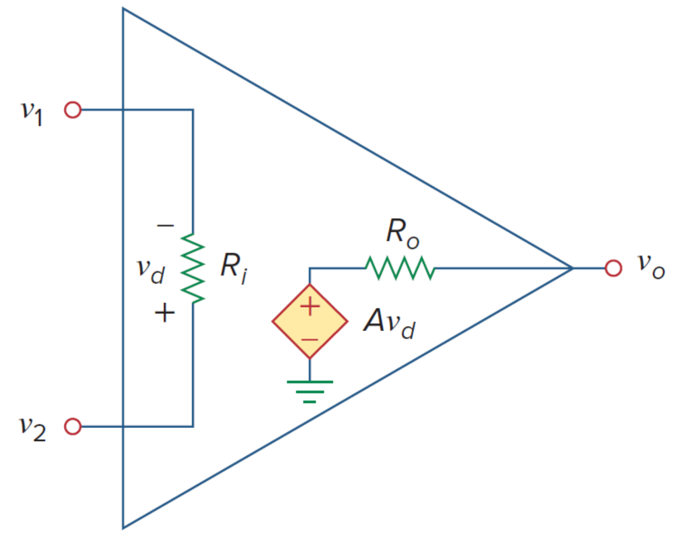


- v_1 is the voltage between the **inverting terminal** and ground and v_2 is the voltage between the **noninverting terminal** and ground. The op amp senses the difference between the two inputs $v_d = v_2 - v_1$, multiplies it by the **op amp open-loop voltage gain** A, and causes the resulting voltage to appear at the output:

$$v_o = Av_d = A(v_2 - v_1)$$

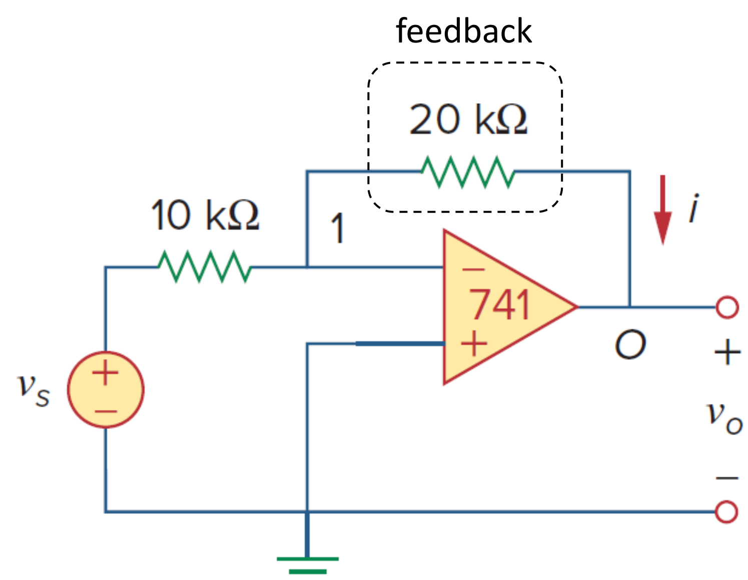
Typical Ranges for Op Amp Parameters

Parameter	Typical range
Open-loop gain, A	10^5 to 10^8
Input resistance, R_i	10^5 to $10^{13}\ \Omega$
Output resistance, R_o	10 to 100 Ω
Supply voltage, V_{CC}	5 to 24 V



Feedback

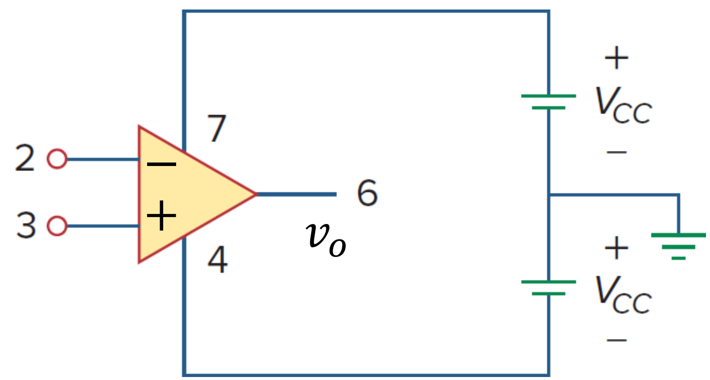
- Op-amps take on an expanded functional ability with the use of feedback. The idea is that the output of the op-amp is fed back into the **inverting terminal**.
- Depending on what elements this signal passes through, the gain and behavior of the op-amp (i.e., the ratio of the output voltage to the input voltage) changes. This ratio is called is called the **closed-loop gain**.
- Feedback to the inverting terminal is called "**negative feedback**".
- Positive feedback would lead to oscillations.

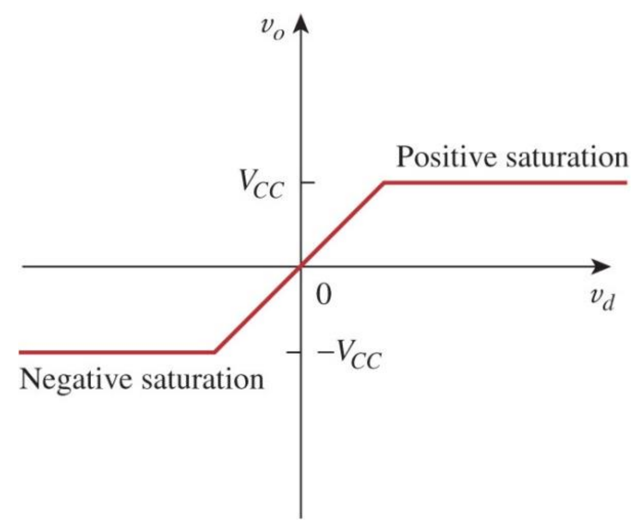


Voltage Saturation

- As an ideal source, the output voltage would be unlimited. However, in reality, one cannot expect the output to exceed the supply voltages V_{CC}.
- When an output should exceed the possible voltage range, the output remains at either the maximum or minimum supply voltage. This is called **saturation**.
- Outputs between these limiting voltages are referred to as the **linear region**.

$$-V_{CC} \leq v_o \leq V_{CC}$$

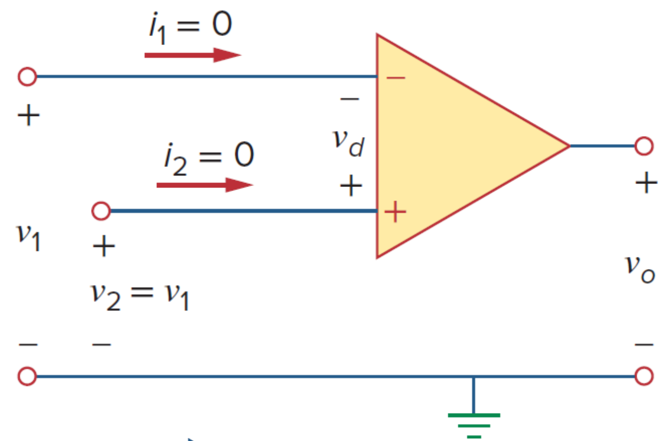




Ideal Op Amp

To facilitate the understanding of op amp circuits, we will assume **ideal op amps**. An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, $A \simeq \infty$.
2. Zero output resistance, $R_o \simeq 0$. Thus, an ideal op amp is load independent.
3. Infinite input resistance, $R_i \simeq \infty$. Thus, for an ideal op amp

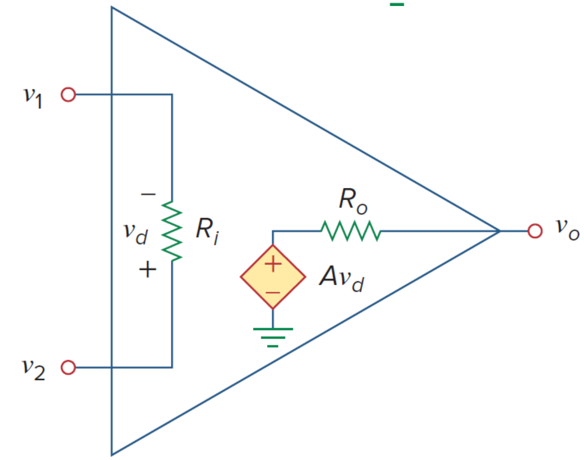


i) The currents into both input terminals are zero.

$$i_1 = 0, \qquad i_2 = 0$$

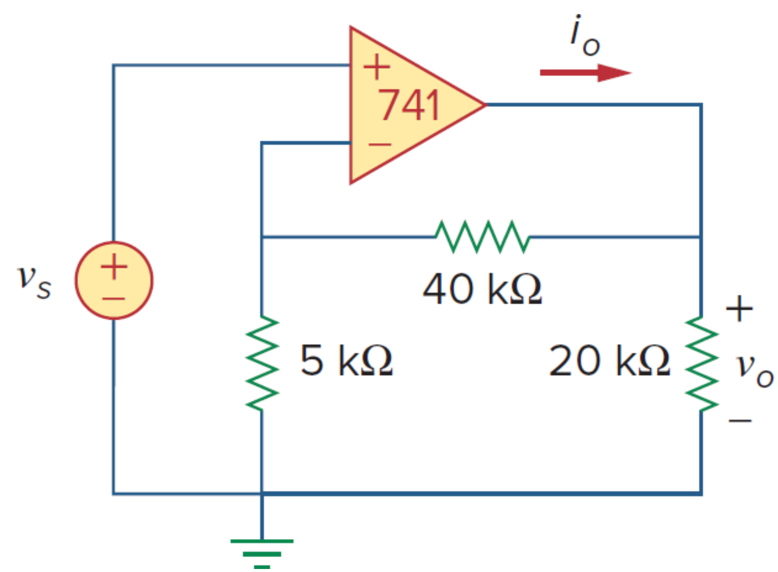
ii) The voltage between the two input terminals is equal to zero.

$$v_d = v_2 - v_1 = 0 \quad \Rightarrow \quad v_2 = v_1$$



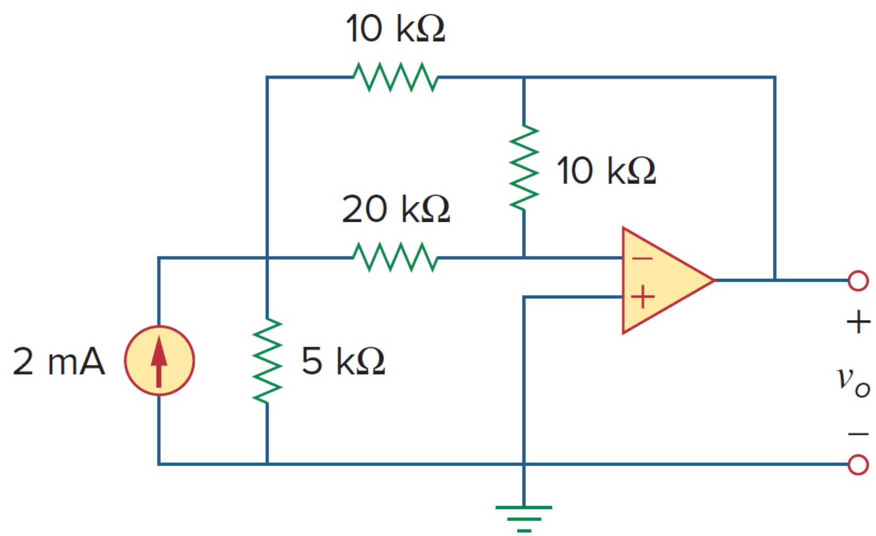
Example

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50 Ω. The op amp is used in the following circuit. **Using the ideal op amp model**, calculate the closed-loop gain v_o / v_s. Find i_o when $v_s = 1$ V.



Example

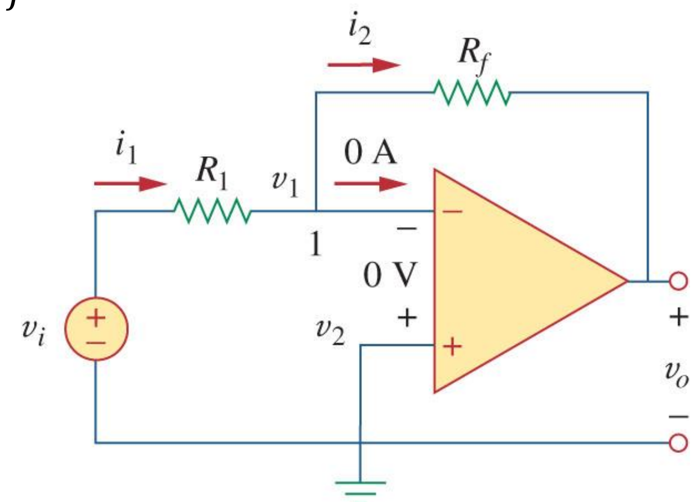
Using the ideal op amp model, determine the output voltage v_o in the circuit.



Inverting and Noninverting Amplifiers

Inverting Amplifier

In an **Inverting Amplifier** circuit, the noninverting input is grounded, v_i is connected to the inverting input through R_1, and the feedback resistor R_f is connected between the inverting input and output.

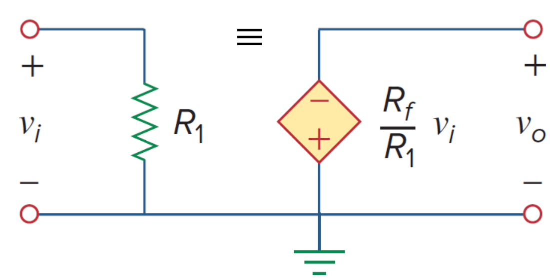


Applying KCL at node 1:

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$

Since the noninverting terminal is grounded, for an ideal op amp, $v_1 = v_2 = 0$. Thus,

$$v_o = -\underbrace{\frac{R_f}{R_1}}_{\text{(Gain)}} v_i$$

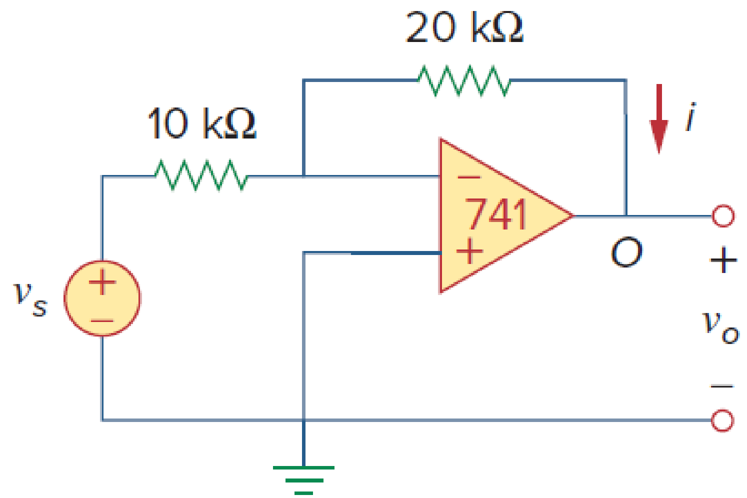


(inverting amplifier's equivalent)

An inverting amplifier reverses the polarity of the input signal while amplifying it.

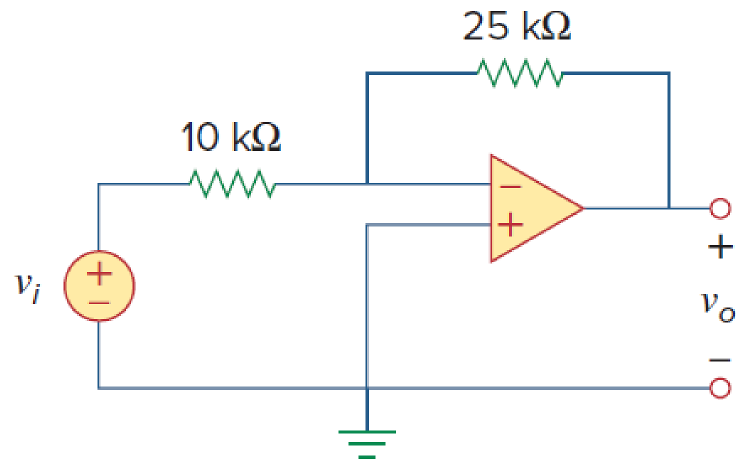
Example

Using the ideal op amp model, find the closed-loop gain $v_o \,/\, v_s$. Determine current i when $v_s = 2$ V.



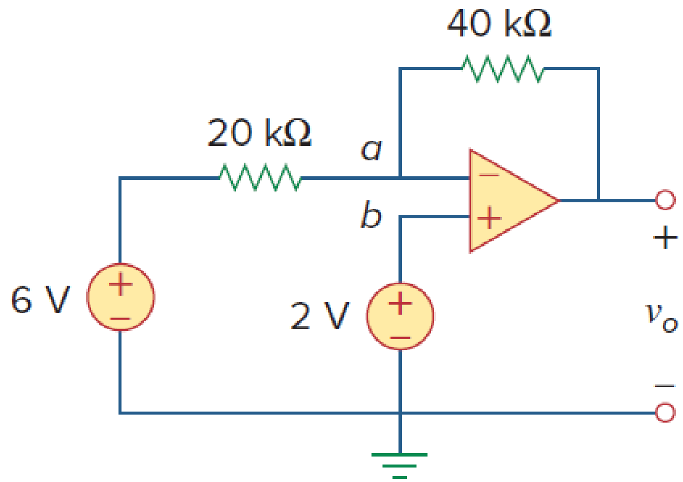
Example

If $v_i = 0.5$ V, calculate: (a) the output voltage v_o, and (b) the current in the 10-kΩ resistor.



Example

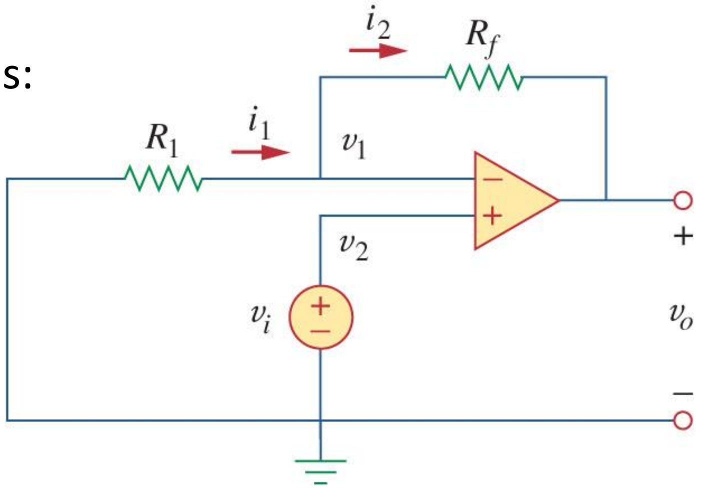
Determine v_o in the op amp circuit shown.



Noninverting Amplifier

- The basic configuration of the **noninverting amplifier** is the same as the inverting amplifier. Except that the input and the ground are switched.
- Once again, applying KCL to the inverting terminal gives:

$$i_1 = i_2 \quad \Rightarrow \quad \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f}$$



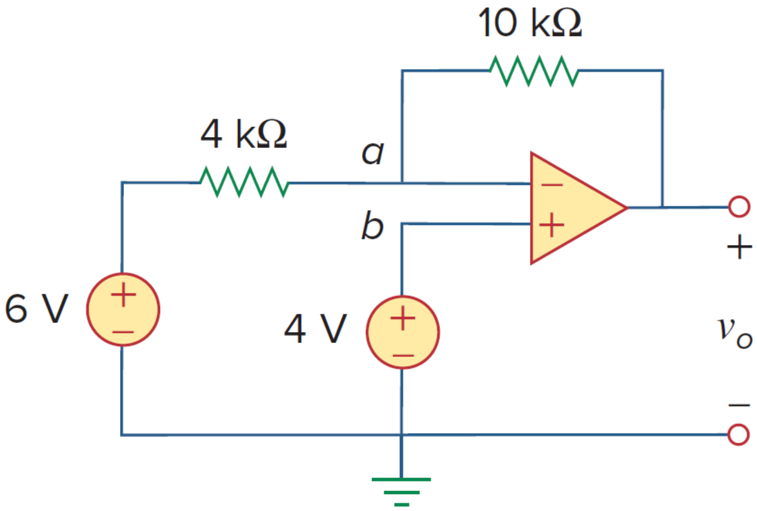
- There is once again negative feedback in the circuit, thus, $v_1 = v_2 = v_i$. This gives:

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f} \quad \Rightarrow \quad v_o = \underbrace{\left(1 + \frac{R_f}{R_1}\right)}_{\text{(Gain)}} v_i$$

Thus, the output has the same polarity as the input, and it can never go below 1.

Example

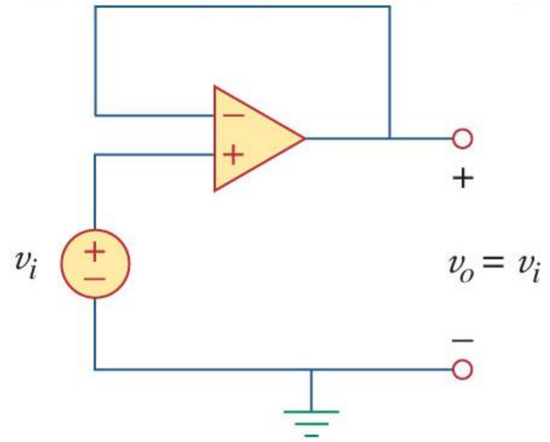
For the op amp circuit in the figure, calculate the output voltage v_o.



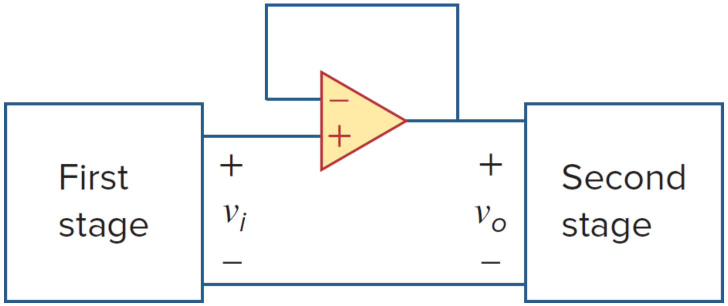
Voltage Follower (or Unity Gain Amplifier)

In a noninverting amplifier, if feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1. Under these conditions, the circuit is called a **voltage follower** (or **unity gain amplifier**) because the output follows the input.

$$v_o = v_i$$



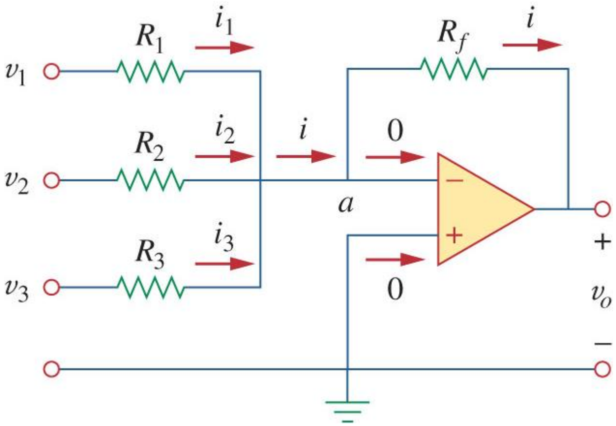
- Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another, while allowing a signal to pass through.
- The voltage follower minimizes interaction between the two stages and eliminates interstage loading.



Summing and Difference Amplifiers

Summing Amplifier

A summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.



Applying KCL at node a (where $v_a = 0$) gives:

$$i = i_1 + i_2 + i_3$$

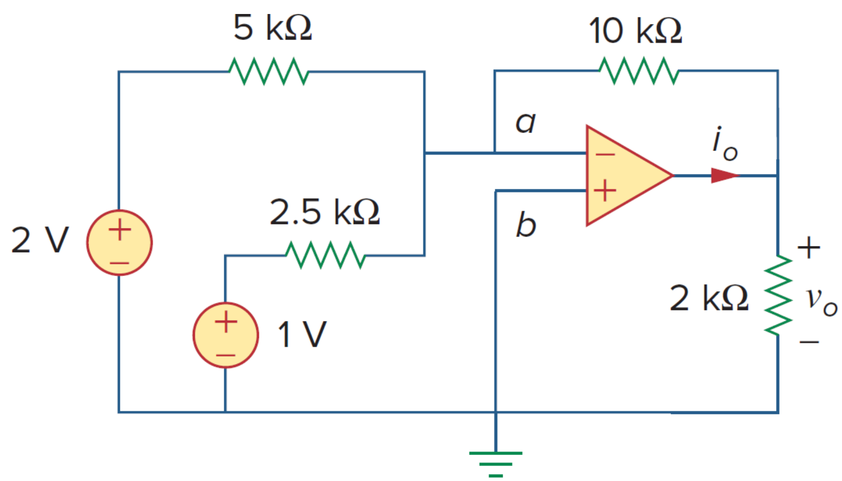
$$i_1 = \frac{v_1 - v_a}{R_1}, i_2 = \frac{v_2 - v_a}{R_2}, i_3 = \frac{v_3 - v_a}{R_3}, i = \frac{v_a - v_o}{R_f}$$

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$

The number of inputs need not be limited to three!

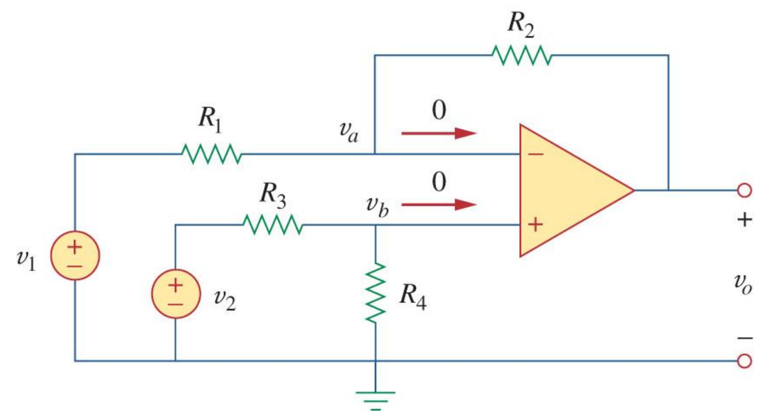
Example

Calculate v_o and i_o in the op amp circuit.



Difference Amplifier

A difference amplifier is a device that amplifies the difference between two inputs and can rejects any signals common to the two inputs.



Applying KCL to node a:

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \quad \Rightarrow \quad v_o = \left(\frac{R_2}{R_1} + 1\right) v_a - \frac{R_2}{R_1} v_1$$

Applying KCL to node b: $$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4} \quad \Rightarrow \quad v_b = \frac{R_4}{R_3 + R_4} v_2$$

Since $v_a = v_b$:

$$v_o = \left(\frac{R_2}{R_1} + 1\right) \frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1 \quad \Rightarrow \quad v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

Common Mode Rejection

- If $\frac{R_1}{R_2} = \frac{R_3}{R_4}$, the difference amplifier becomes $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

In this case, when $v_1 = v_2$, then $v_o = 0$ (i.e., a difference amplifier can reject a signal common to the two inputs).

- If $R_2 = R_1$ and $R_3 = R_4$, the difference amplifier becomes a **subtractor**:

$$v_o = v_2 - v_1$$

Example

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

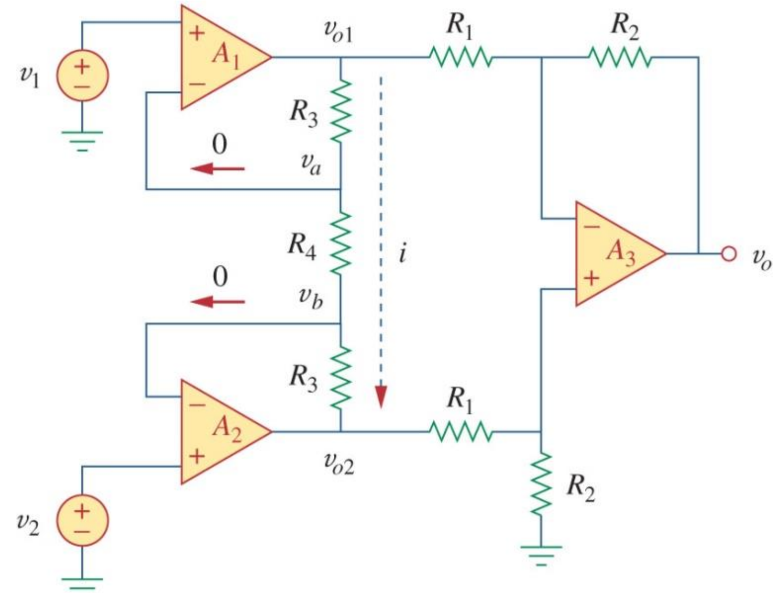
Design 1) Use only one op amp and at least one 10-KΩ resistor.

Design 2) Use two op amps and at least one 10-KΩ resistor.

Instrumentation Amplifier

The difference amplifier has one significant drawback: The input impedance is low. By placing 2 noninverting amplifiers stage before the difference amplifier this can be resolved.

An **instrumentation amplifier** is an amplifier of low-level signals used in process control or measurement applications and commercially available in single-package units.



Since the amplifier A_3 is a difference amplifier:

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1})$$

Since the op amps A_1 and A_2 draw no current, current i flows through the three resistors as though they were in series.

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4)$$

$$i = \frac{v_a - v_b}{R_4} \xrightarrow{v_a = v_1, v_b = v_2} i = \frac{v_1 - v_2}{R_4}$$

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Integrator and Differentiator Amplifiers

Integrator

- Capacitors, in combination with op-amps can be made to perform advanced mathematical functions.
- One such function is the integrator. An **integrator** is an op amp circuit whose output is proportional to the integral of the input signal.

At node a, $i_R = i_C$

$$i_R = \frac{v_i - v_a}{R}, \qquad i_C = C\frac{d(v_a - v_o)}{dt}$$

$v_a = 0$ →

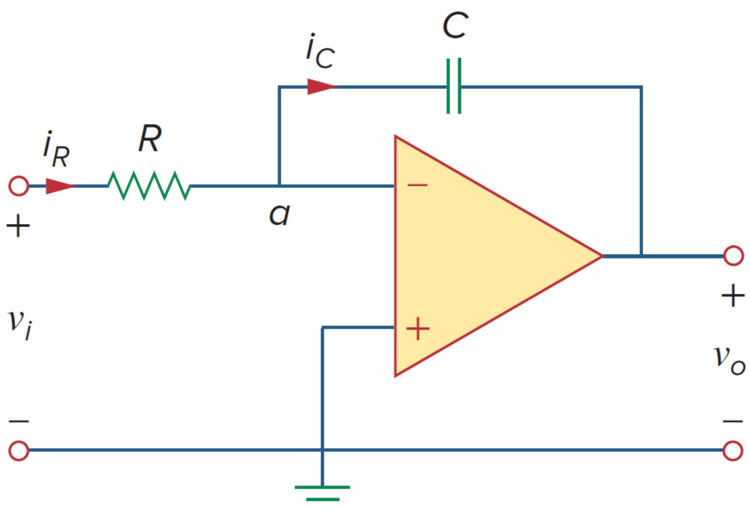
$$\frac{v_i}{R} = -C\frac{dv_o}{dt}$$

$$dv_o = -\frac{1}{RC}v_i dt$$

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau$$

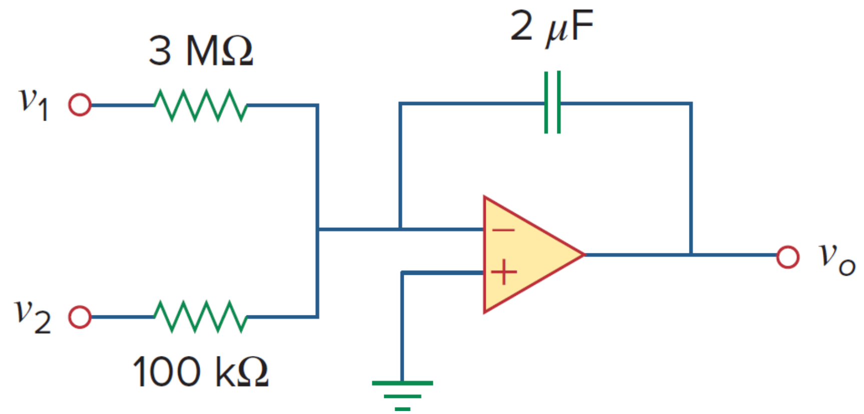
Assuming $v_o(0) = 0$,

$$v_o = -\frac{1}{RC}\int_0^t v_i(\tau)d\tau$$



Example

If $v_1 = 10\cos 2t$ mV and $v_2 = 0.5t$ mV, find v_o in the op amp circuit. Assume that the voltage across the capacitor is initially zero.



Differentiator

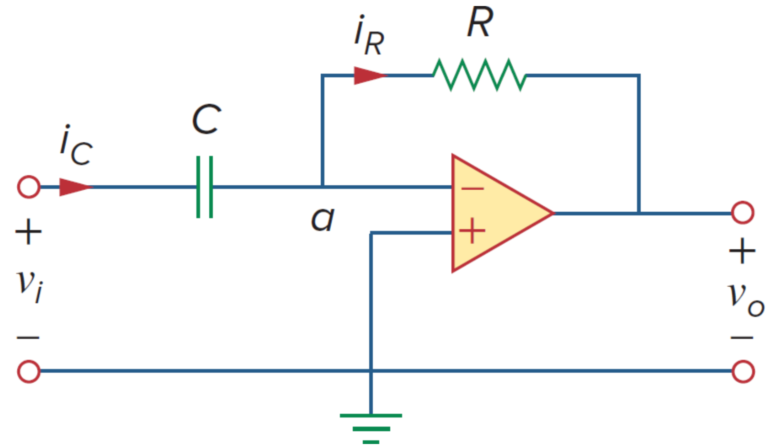
A **differentiator** is an op amp circuit whose output is proportional to the rate of change of the input signal.

At node a, $i_R = i_C$

$$i_R = \frac{v_a - v_o}{R}, \qquad i_C = C\frac{d(v_i - v_a)}{dt}$$

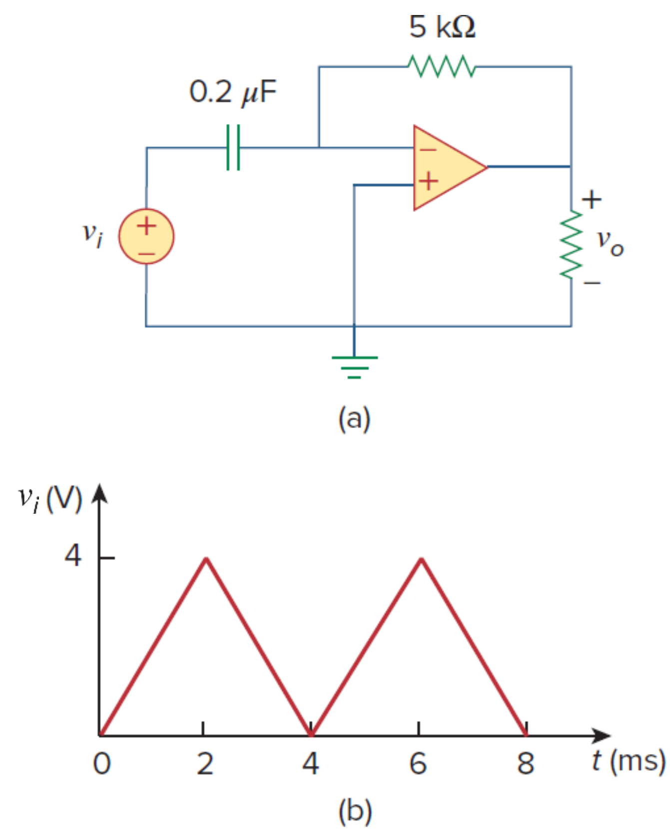
$v_a = 0$ →

$$v_o = -RC\frac{dv_i}{dt}$$



Example

Sketch the output voltage for the circuit, given the input voltage. Take $v_o = 0$ at $t = 0$.



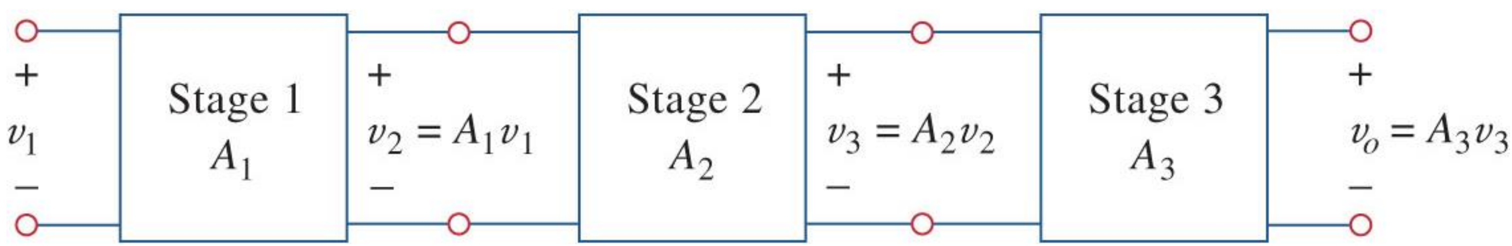
Cascaded Op Amp Circuits

Cascaded Op Amps

A **cascade connection** is a head-to-tail arrangement of two or more op amp circuits such that the output of one is the input of the next.

- It is often necessary in practical applications to connect op amp circuits in cascade (i.e., head to tail) to achieve a large overall gain. Each amplifier in the string is called a **stage**.
- Op amp circuits have the advantage that they can be cascaded without changing their input-output relationships. This is due to the fact that each (ideal) op amp circuit has infinite input resistance and zero output resistance.

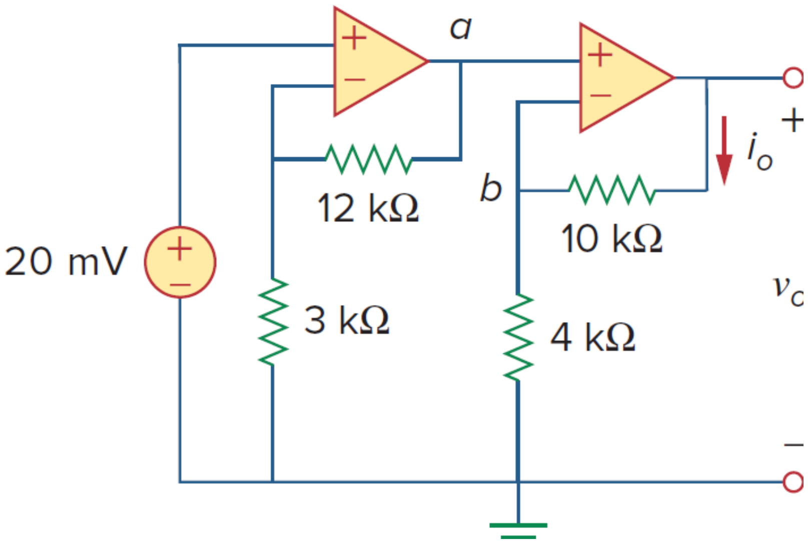
Example: A three-stage cascaded connection.



$$A = A_1 A_2 A_3$$

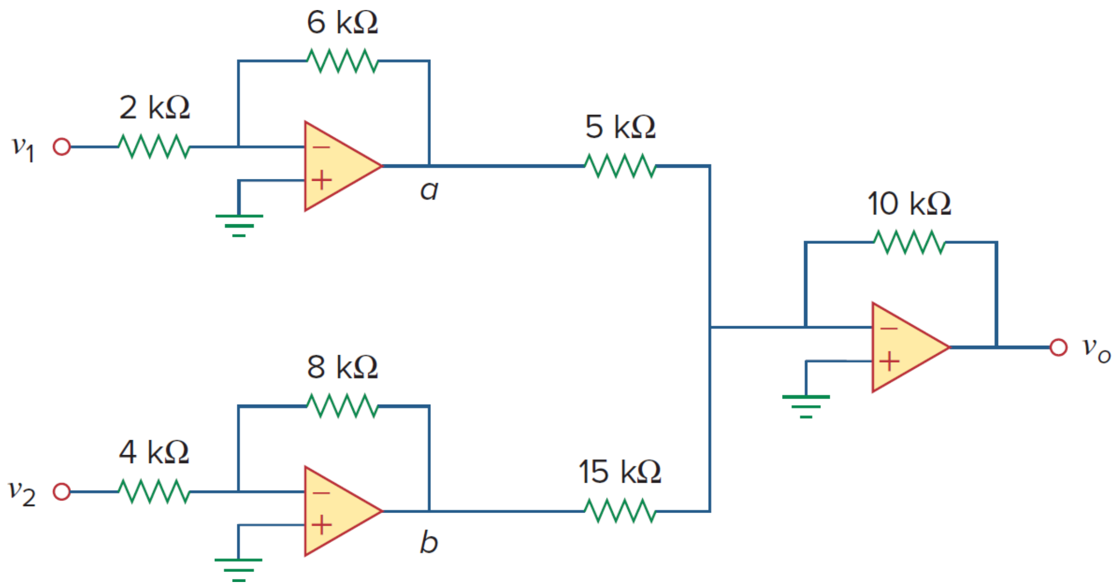
Example

Find v_o and i_o in the circuit.

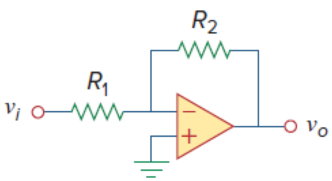
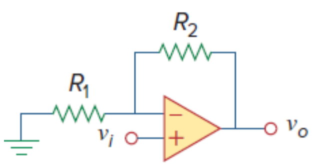
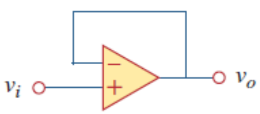
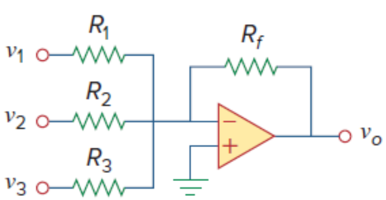
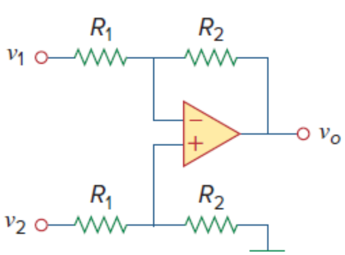


Example

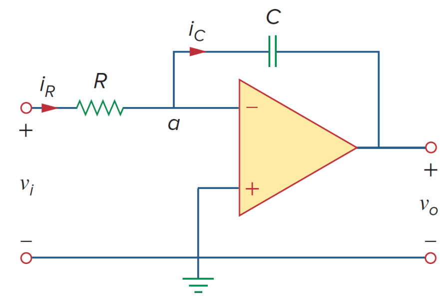
If $v_1 = 1$ V and $v_2 = 2$ V, find v_o in the op amp circuit.



Summary of Basic Op Amp Circuits

Op amp circuit	Name/output-input relationship
	Inverting amplifier $v_o = -\frac{R_2}{R_1} v_i$
	Noninverting amplifier $v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$
	Voltage follower $v_o = v_i$
	Summer $v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$
	Difference amplifier $v_o = \frac{R_2}{R_1}(v_2 - v_1)$

Integrator:



Differentiator: